Universal Law of Gravitation

Law: Every body exerts a force of attraction on every other body. This force called, **gravity**, is relatively weak and decreases rapidly with the distance separating the bodies but increases with the masses of the bodies. I.e.,

$$F_g = G\frac{m_1 m_2}{r^2}$$

On earth, the effects of these forces on ourselves or on the objects we use daily are too small to be of any consequence. The exceptions are the forces produced by the earth's gravitation on all the bodies, including ourselves, that are in its immediate vicinity. This force, called **weight**, acting on a mass, *m*, is equal to:

$$W = m \times G\frac{m_{earth}}{r_{earth}^2} = mg$$

where, ***g***, is the acceleration caused by earth's gravitational pull. Unlike the **Universal Gravitational Constant (*G*)**, *g* varies slightly with altitude and latitude because these variables influence the distance, *r*, between the centre of the earth and the mass, *m*. For most purposes a value for *g* of 9.81 m/s^2 is sufficiently accurate. The direction of this force is downwards towards the centre of mass of the earth.

To determine a body's weight multiply the body's mass in kilograms by 9.81 to obtain its weight in newtons.

$$W = 65\ kg \times 9.81\ m/s^2$$

$$= 637.65\ kg.m/s^2 = 638\ N$$

A **newton (N)** is defined as the force required to accelerate a 1 kilogram mass at the rate of 1 m/s^2.

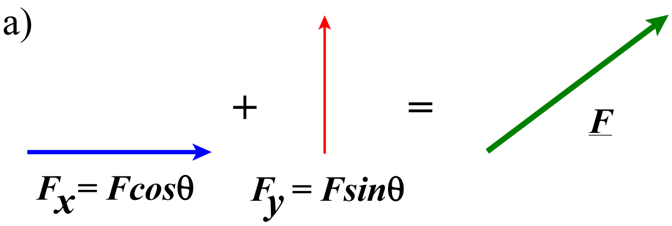
Resolution of a Force

Adding vectors, specifically forces, graphically using the **parallelogram law** (or the triangle or polygon rules) is very cumbersome. An easier method is to resolve each vector into its components and then adding the components algebraically. To resolve a vector into its components we apply trigonometry.

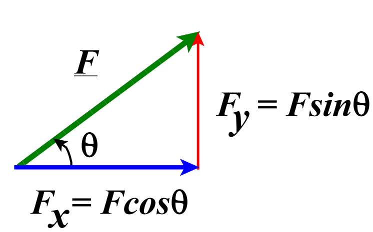
That is,

$$\underline{F} = (F_x, F_y) = (F\cos\theta, F\sin\theta)$$

a)



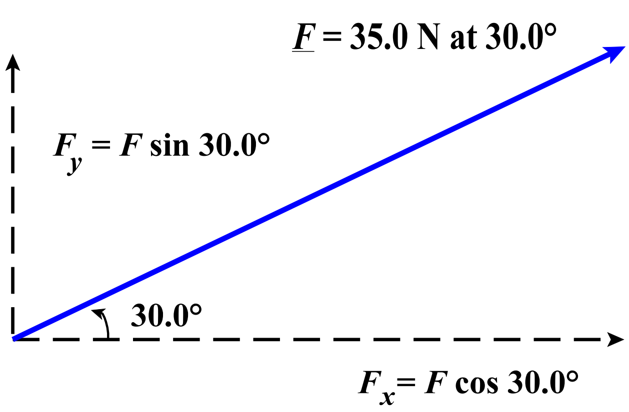
b)



Example:

Compute the components of a 35.0 N force that makes a 30^0 angle to the right horizontal. It helps to draw a diagram.

Therefore,



$$F_x = F\cos 30^\circ = 35 \times 0.866 = 30.3\ N$$
$$F_y = F\sin 30^\circ = 35 \times 0.500 = 17.50\ N$$

Adding Forces by Summing Components

To add a series of forces numerically, resolve each force into its components and sum the components. That is,

$$\underline{F} = \underline{F}_1 + \underline{F}_2 + \underline{F}_3$$

or

$$F_x = F_{1_x} + F_{2_x} + F_{3_x}$$
$$F_y = F_{1_y} + F_{2_y} + F_{3_y}$$
$$(F_z = F_{1_z} + F_{2_z} + F_{3_z})$$

Example: Find the sum of the following forces:

$\underline{\boldsymbol{F}}_{\boldsymbol{a}}$ = 10.00 newtons as 35.0 deg

$\underline{\boldsymbol{F}}_{\boldsymbol{b}}$ = 25.3 newtons at 125.0 deg

$\underline{\boldsymbol{R}}_{\boldsymbol{a+b}} = (R_x, R_y)$

R_x = 10.00 cos 35.0 + 25.3 cos 125.0 = 8.192 –14.511 = –6.319

R_y = 10.00 sin 35.0 + 25.3 sin 125.0 = 5.736 + 20.72 = 26.46

Thus, $\underline{\boldsymbol{R}}_{\boldsymbol{a+b}}$ = (–6.32, 26.5) newtons

Example: Find the sum of the following forces:

$\underline{\boldsymbol{F}}_{\boldsymbol{a}}$ = (10.30, 12.05)

$\underline{\boldsymbol{F}}_{\boldsymbol{b}}$ = (20.5, –10.80)

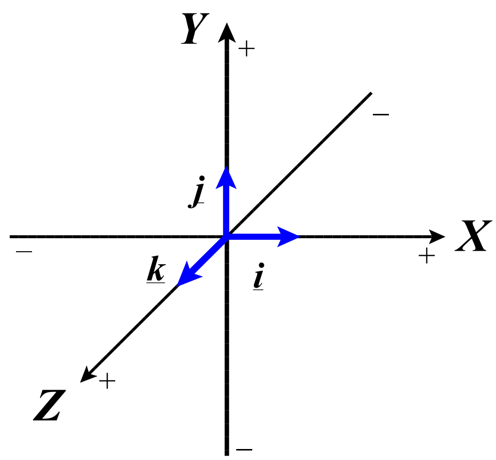
$\underline{\boldsymbol{F}}_{\boldsymbol{c}}$ = (–8.25, –7.20)

$\underline{\boldsymbol{R}}_{\boldsymbol{a+b+c}}$ = (10.30 + 20.5 – 8.25, 12.05 – 10.80 – 7.20)
= (22.55, –5.95)

Thus, $\underline{\boldsymbol{R}}_{\boldsymbol{a+b+c}}$ = (22.6, –5.95) newtons

Rectangular Components and the Unit Vectors

Unit vectors: The unit vectors, $\underline{\boldsymbol{i}}$, $\underline{\boldsymbol{j}}$ and $\underline{\boldsymbol{k}}$, are vectors with length 1 (unity) and each possesses a positive direction along one of the three Cartesian axes, X, Y and Z, respectively.



Therefore another way of writing a force vector is:

$$\underline{\boldsymbol{F}} = (F_x, F_y) = F_x \underline{\boldsymbol{i}} + F_y \underline{\boldsymbol{j}}$$

The weight vector ($\underline{\boldsymbol{W}}$) for a weight of 50.0 N would be:

$$\underline{\boldsymbol{W}} = -50.0 \underline{\boldsymbol{j}}\ N$$

Examples:

$\underline{\boldsymbol{R}}_{a+b} = (-6.32, 26.5) = -6.32\ \underline{\boldsymbol{i}} + 26.5\ \underline{\boldsymbol{j}}$ newtons

$\underline{\boldsymbol{R}}_{a+b+c} = (22.6, -5.95) = 22.6\ \underline{\boldsymbol{i}} - 5.95\ \underline{\boldsymbol{j}}$ newtons

Find the sum (i.e., resultant) of the above two forces.

$$\underline{\boldsymbol{R}} = \underline{\boldsymbol{R}}_{a+b} + \underline{\boldsymbol{R}}_{a+b+c} = (-6.32 + 22.6)\ \underline{\boldsymbol{i}} + (26.5 - 5.95)\ \underline{\boldsymbol{j}}$$
$$= 16.28\ \underline{\boldsymbol{i}} + 20.6\ \underline{\boldsymbol{j}} \text{ newtons}$$

Equilibrium of a Particle

Resultant force is the vector sum of all external forces acting on a particle or rigid body. Expressed mathematically as:

$$\Sigma \underline{\boldsymbol{F}} = \underline{\boldsymbol{F}}_1 + \underline{\boldsymbol{F}}_2 + \underline{\boldsymbol{F}}_3 + \ \ldots$$

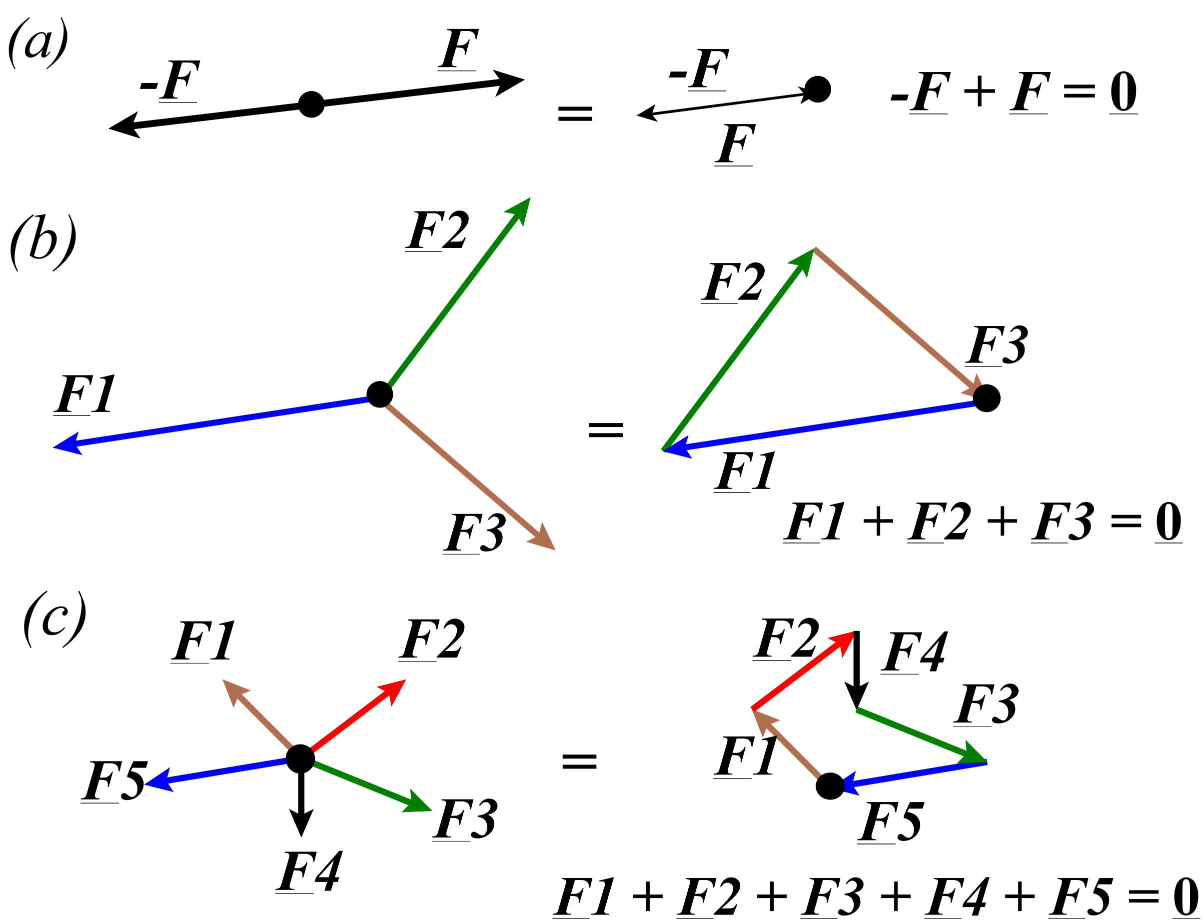
or

$$\Sigma F_x = F_{x_1} + F_{x_2} + F_{x_3} + \ \ldots$$

$$\Sigma F_y = F_{y_1} + F_{y_2} + F_{y_3} + \ \ldots$$

$$(\Sigma F_z = F_{z_1} + F_{z_2} + F_{z_3} + \ \ldots)$$

Graphically this concept may be expressed by force polygons that are **closed**. That is,



Newton's First Law: Law of Inertia

Static Equilibrium: When the resultant force acting on a particle is equal to zero, the particle is in **static equilibrium**. A particle in static equilibrium is governed by Newton's First Law which states:

> "*Corpus omne perseverare in statuo suo quiescendi vel movendi uniformiter in directum, nisi quatenus a viribus impressis cogitur statum illum mutare.*"

Or in English

> "Every body continues in its state of rest or of uniform motion in a straight line, unless it is compelled to change that state by forces impressed upon it."

Using the concept of the **resultant force**:

> "A body will remain at rest or in constant linear motion whenever the **resultant force** acting upon it is zero."

Or mathematically:

$$\Sigma \underline{\boldsymbol{F}} = \underline{\boldsymbol{0}}$$

or

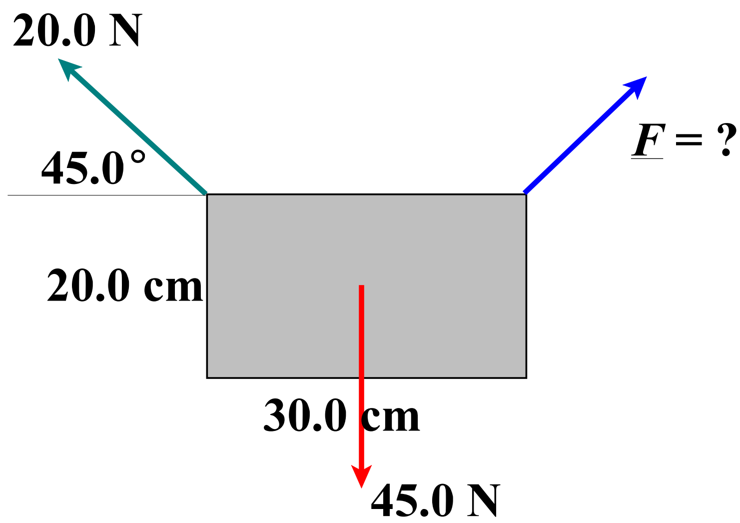
$$\Sigma F_x = 0$$
$$\Sigma F_y = 0$$
$$(\Sigma F_z = 0)$$

The converse of this law also applies. In other words, a body that is at rest or in constant linear motion must have a resultant force equal to zero.

It is therefore possible to compute an unknown force whenever all other forces acting upon a body are known. For example, a 600 N person who is standing motionless on a diving board must be experiencing a vertical reaction force from the board equal to 600 N.

Example:

If an object is suspended as shown below, what are the components of the force, $\underline{\boldsymbol{F}}$, that keeps the object in static equilibrium?



Since, $\underline{\boldsymbol{F}} = \underline{\mathbf{0}}$ then $F_x = 0$ and $F_y = 0$

Therefore, $F_x - 20.0 \cos 45° = 0$
$F_x = 20.0 \cos 45° = 14.142$

and $F_y + 20.0 \sin 45° - 45.0 = 0$
$F_y = 45.0 - 20.0 \sin 45° = 30.86$

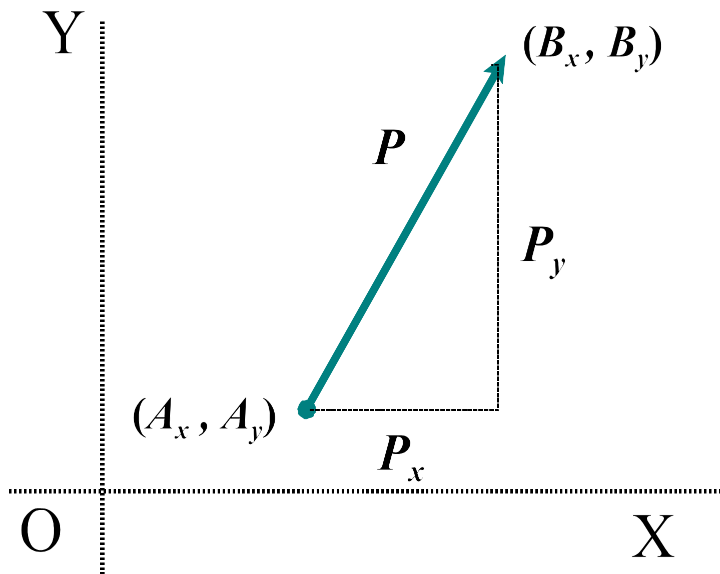
Thus, the force necessary for static equilibrium is:

$\underline{\boldsymbol{F}} =$ (14.14, 30.9) newtons.

Linear Position Vectors

These are vectors that connect one position to another as shown in the following figure.

Position vectors are usually written $\overline{AB}$ which means the vector starts at point *A* and ends at point *B.*



To define the vector in rectangular coordinates notation, subtract the coordinates of the first position (*A*) from those of the second position (*B*).

That is: $\underline{\boldsymbol{P}} = (B_x - A_x, B_y - A_y)$

Example: What is the position vector, $\overline{AB}$, if $A = (3.00, 1.00)$ and $B = (6.00, 4.50)$?

$$\overline{AB} = (6.00 - 3.00, 4.50 - 1.00) = (3.00, 3.50)$$

What is the position vector, $\overline{BA}$?

$$\overline{BA} = (3.00 - 6.00, 1.00 - 4.50) = (-3.00, -3.50)$$

Note: $\overline{AB} = -\overline{BA}$

Angular Positions

Angular Units of Measure

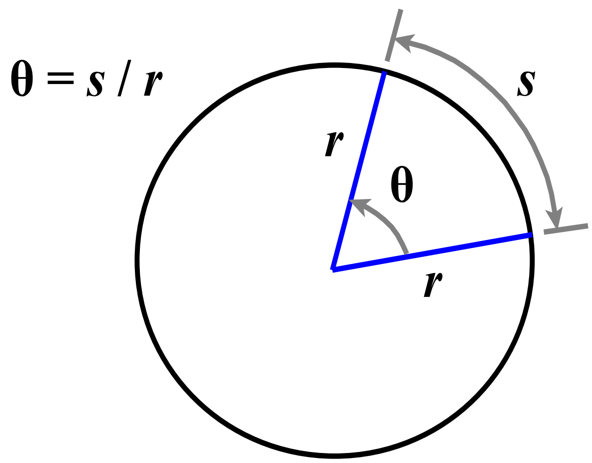
First at least three different SI units of measure are permissible. They are degrees, radians and revolutions, abbreviated deg, rad and r, respectively. They are related in the following way:

1 revolution = 360 degrees = 2   radians

Radian measure is considered the preferred measure since, by definition, it is a dimensionless measure. The **radian** is defined as the angle formed when an arc length equals the radius of a circle. Thus, angles may be described as arc length (***s***) divided by radius (***r***). I.e.,

= s / r

The concept of pi ( ) is employed because since ancient times this value was defined as the ratio of the circumference of a circle to its diameter. Since a diameter is equivalent to going half way around the circle, there are 2   radians in 360 degrees.



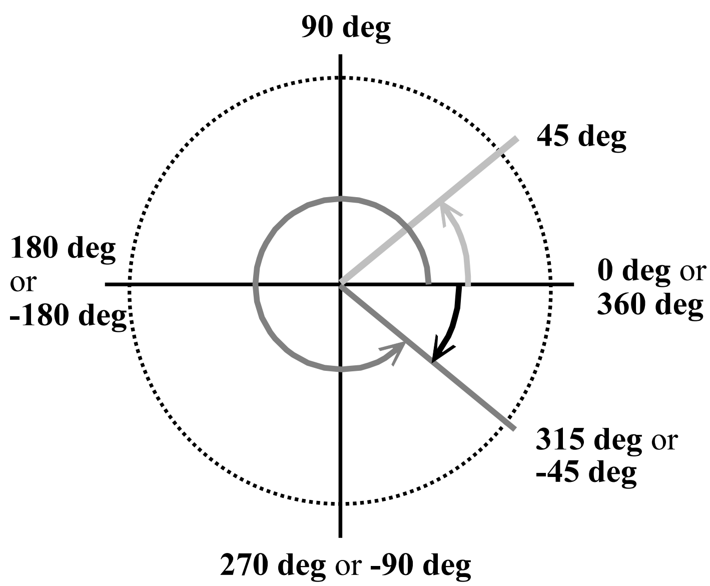
When $s = r$, $\theta = 1$ radian

Angular Positions

The angular position of a line segment may be defined in a number of different ways. Once the a particular unit of measure is selected the next choice for defining the angular position of a line segment is which axis to use as a reference. Generally it is accepted to use the positive X-axis but any axis could be used.

Reference Axes

In addition, one could choose to use a system that defines angles between 0 and 360 degrees or one that goes from –180 to +180 degrees. The diagram at right shows these two systems. In this diagram the angles were measured from the positive X-axis.



Note that angular measures are not vectors because they do not add according to the Parallelogram Law (i.e., they are not commutative) since + does not necessarily equal , as shown on the right.