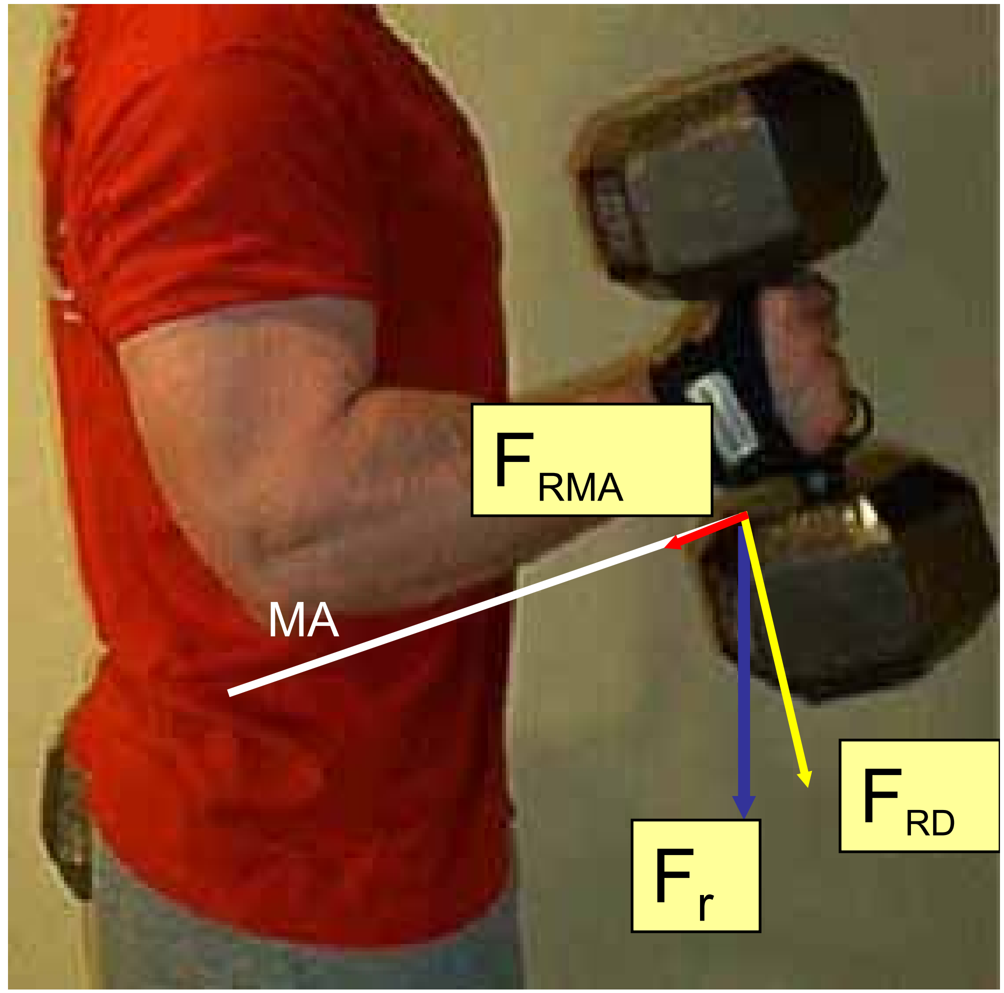
Section 6: Kinematics

Biomechanics - angular kinematics

- Same as linear kinematics, but…
- There is one vector along the moment arm.
- There is one vector perpendicular to the moment arm.



From: Legh

Translational vs Rotational

- Linear momentum
  =
  mass $\times$ velocity

- d/dt (linear momentum) = applied forces

- d/dt (position)
  =
  linear momentum/mass

- Angular momentum
  =
  inertia $\times$ angular velocity

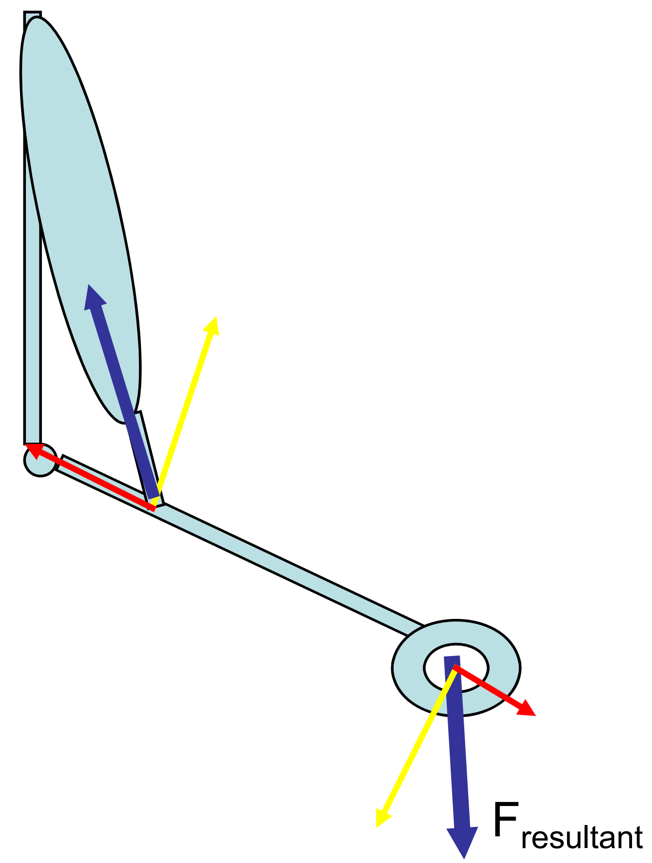
- d/dt (angular momentum)
  =
  applied torques

- d/dt (attitude)
  =
  "angular momentum/inertia"

From: Hall

Vectors

- Remember, Vectors are representative of the MAGNITUDE of a resultant FORCE



From: Legh

Vectors

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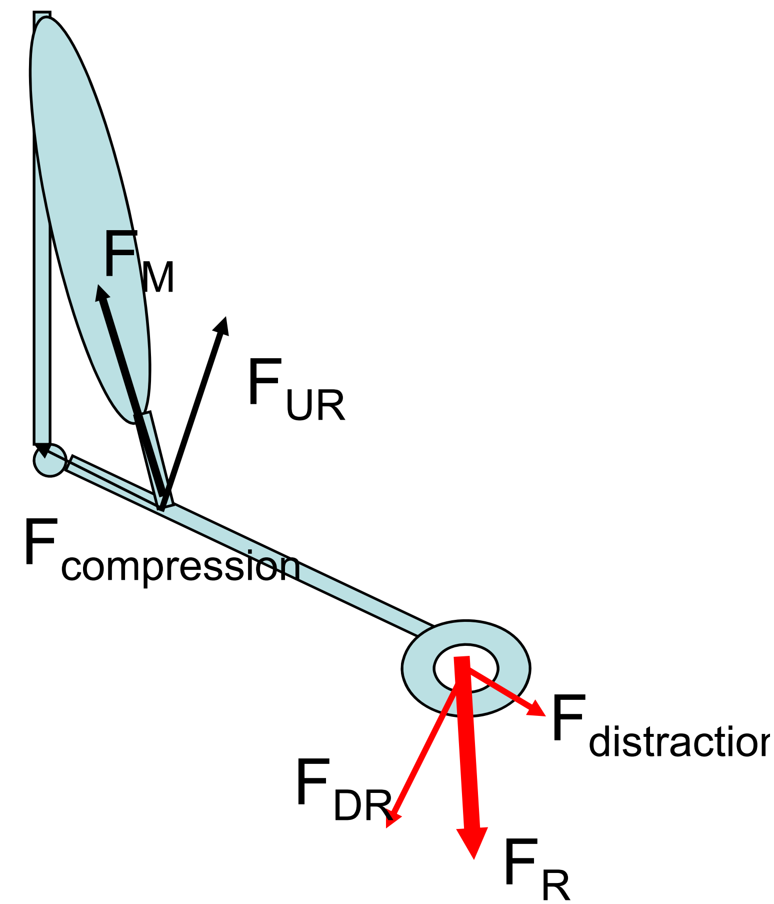


From: Legh

Vectors

- A vector is an abstract mathematical object with two properties: length or magnitude. and direction

$$\vec{\mathbf{v}} = v_1 \hat{\mathbf{i}}_1 + v_2 \hat{\mathbf{i}}_2 + v_3 \hat{\mathbf{i}}_3 \qquad (3.14)$$

$$v_1 = \vec{\mathbf{v}} \cdot \hat{\mathbf{i}}_1, \quad v_2 = \vec{\mathbf{v}} \cdot \hat{\mathbf{i}}_2, \quad v_3 = \vec{\mathbf{v}} \cdot \hat{\mathbf{i}}_3 \quad (3.15)$$

$$v_1 = v \cos \alpha_1, \quad v_2 = v \cos \alpha_2, \quad v_3 = v \cos \alpha_3$$

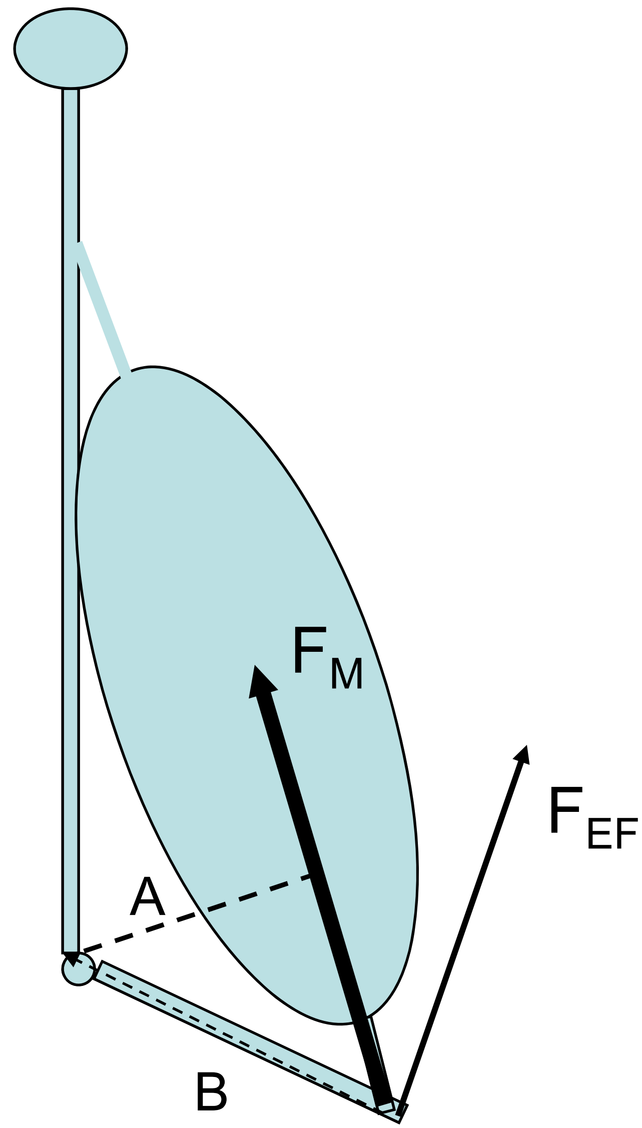
$$\mathbf{v} = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} \qquad \vec{\mathbf{v}} = [v_1 \quad v_2 \quad v_3] \begin{Bmatrix} \hat{\mathbf{i}}_1 \\ \hat{\mathbf{i}}_2 \\ \hat{\mathbf{i}}_3 \end{Bmatrix} = \mathbf{v}_i^T \left\{ \hat{\mathbf{i}} \right\}$$

$$\mathbf{v}_i, \mathbf{v}_o, \mathbf{v}_b \qquad \vec{\mathbf{v}} = \mathbf{v}_i^T \left\{ \hat{\mathbf{i}} \right\} = \mathbf{v}_o^T \left\{ \hat{\mathbf{o}} \right\} = \mathbf{v}_b^T \left\{ \hat{\mathbf{b}} \right\} \qquad (3.19)$$

From: Hall

Moment Arm

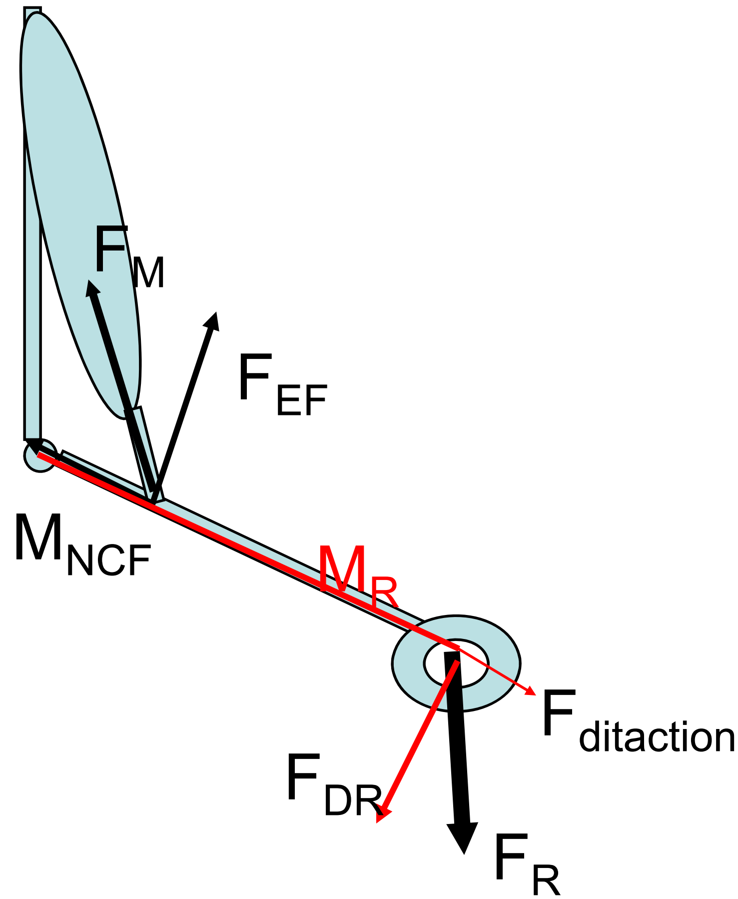
- The MOMENT ARM (M) is the perpendicular distance from the line of resultant force to the fulcrum (joint axis), A, or the distance from axis of rotation to the point of muscle insertion, B.



From: Legh

Torque

- Torque, or rotational force, is a product of the rotational component(F_{ur}) x the moment arm, or the resultant force of muscular contraction (F_{M}) x perpendicular distance from F_M to axis of rotation.



From: Legh

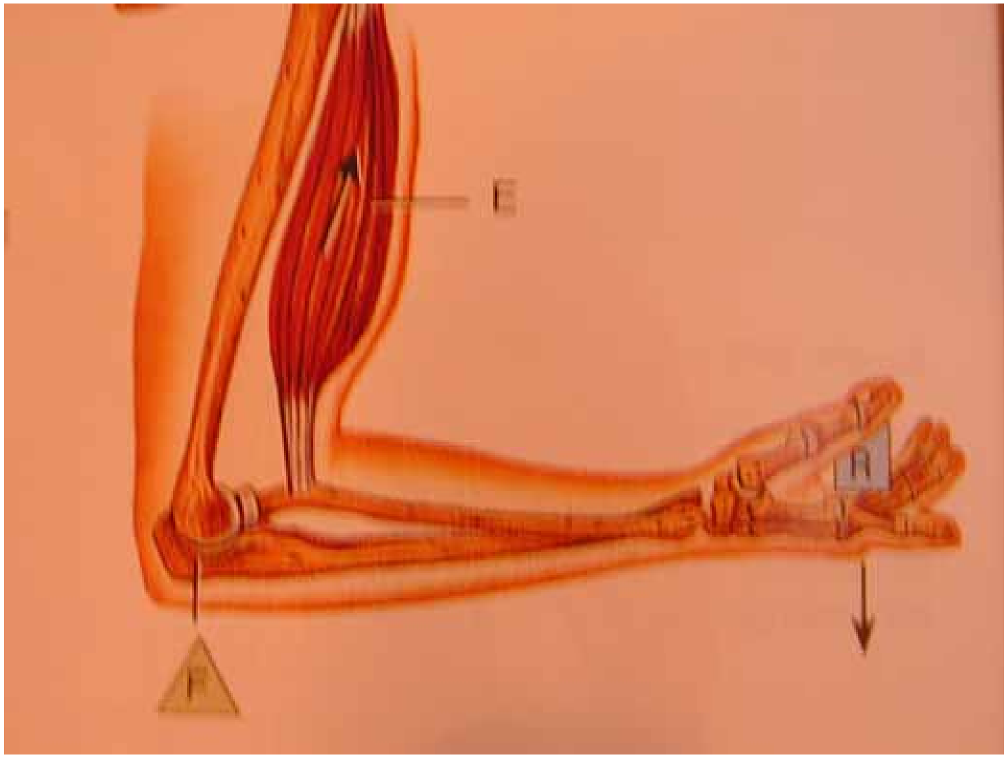
Biomechanics

Class III Lever

The muscular force is between the fulcrum and the resistance force.

The most common.

The least efficient.



From: Legh

Angular Kinematic Analysis

- Angular Kinematics
  - Description of the circular motion or rotation of a body
- Motion described in terms of (variables):
  - Angular position and displacement
  - Angular velocity
  - Angular acceleration
- Rotation of body segments
  - e.g. Flexion of forearm about transverse axis through elbow joint centre
- Rotation of whole body
  - e.g. Rotation of body around centre of mass (CM) during somersaulting

From: Biolab

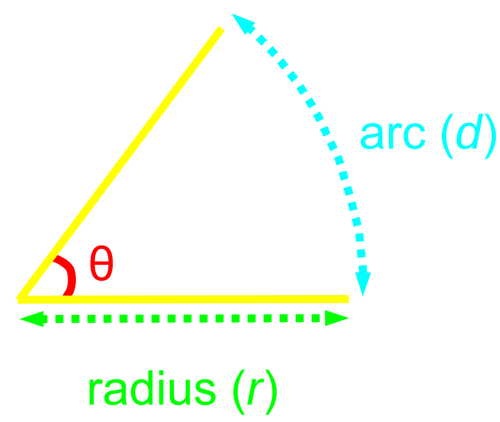
Absolute and Relative Angles

- Absolute angles
  - Angle of a single body segment, relative to (normally) a right horizontal line (e.g. trunk, head, thigh)
- Relative Angles
  - Angle of one segment relative to another (e.g. knee, elbow, ankle)

From: Biolab

Units of Measurement

- Angles are expressed in one of the following units:
- Revolutions (Rev)
  - Normally used to quantify body rotations in diving, gymnastics etc.
  - 1 rev = 360º or 2 π radians
- Degrees (º)
  - Normally used to quantify angular position, distance and displacement
- Radians (Rad)
  - Normally used to quantify angular velocity and acceleration
  - Convert degrees to radians by dividing by 57.3



$$\theta = \frac{d}{r} = 1 \text{ radian}$$

From: Biolab

Method of Problem Solution

- *Problem Statement*:
  Includes given data, specification of what is to be determined, and a figure showing all quantities involved.
- *Free-Body Diagrams*:
  Create separate diagrams for each of the bodies involved with a clear indication of all forces acting on each body.
- *Fundamental Principles*:
  The six fundamental principles are applied to express the conditions of rest or motion of each body. The rules of algebra are applied to solve the equations for the unknown quantities.
- *Solution Check*:
  - Test for errors in reasoning by verifying that the units of the computed results are correct,
  - test for errors in computation by substituting given data and computed results into previously unused equations based on the six principles,
  - **always** apply experience and physical intuition to assess whether results seem "reasonable"

Free Body Diagrams

- Space diagram represents the sketch of the physical problem. The free body diagram selects the significant particle or points and draws the force system on that particle or point.
- **Steps:**
- **1.** Imagine the particle to be isolated or cut free from its surroundings. Draw or sketch its outlined shape.

Free Body Diagrams Contd.

- 2. Indicate on this sketch all the forces that act on the particle.
- These include active forces - tend to set the particle in motion e.g. from cables and weights and reactive forces caused by constraints or supports that prevent motion.

Free Body Diagrams Contd.

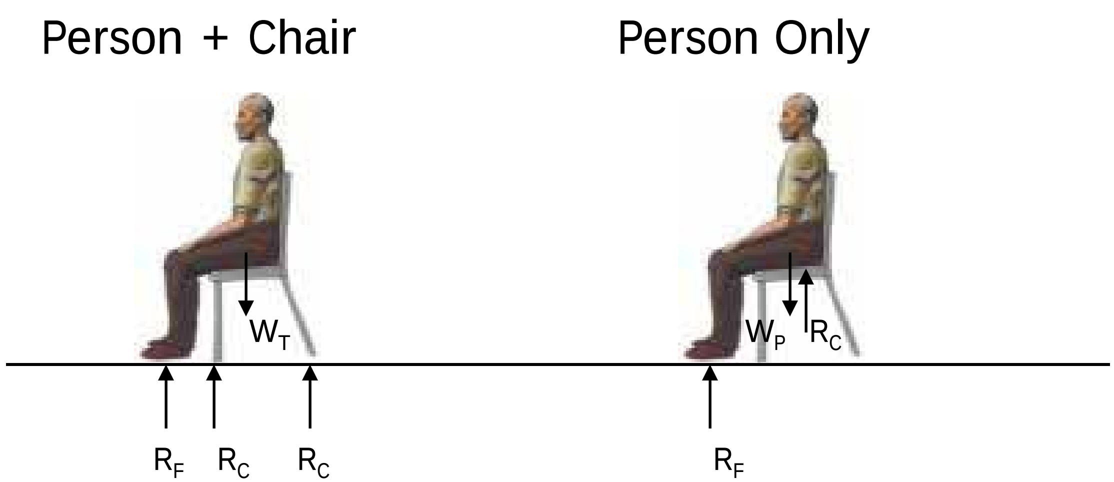
- 3. Label known forces with their magnitudes and directions. use letters to represent magnitudes and directions of unknown forces.
- Assume direction of force which may be corrected later.

Free Body Diagrams

- Most important analysis tool
- Aids in identification of external forces
- Procedure
  - Identify the object to be isolated
  - Draw the object isolated (with relevant dimensions)
  - Draw vectors to represent all external forces

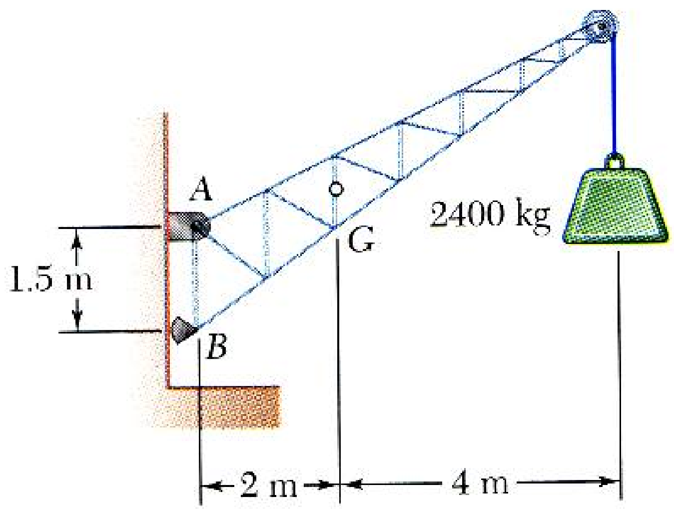
Free Body Diagrams

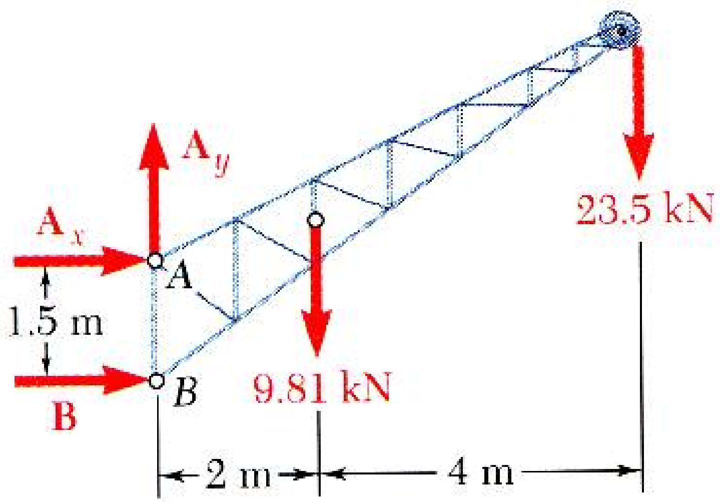
- Internal/External Force
  - Depends on choice of object



From: Gabauer

Free-Body Diagram



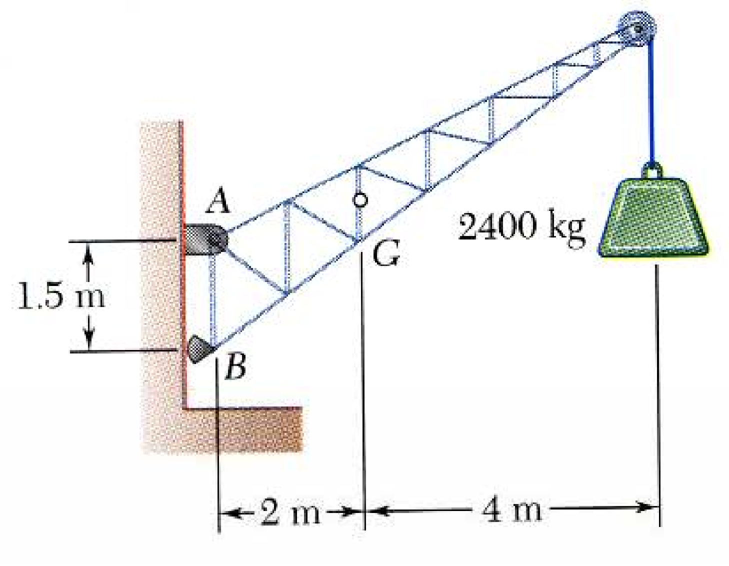


First step in the static equilibrium analysis of a rigid body is identification of all forces acting on the body with a *free-body* diagram.

- Select the extent of the free-body and detach it from the ground and all other bodies.
- Indicate point of application, magnitude, and direction of external forces, including the rigid body weight.
- Indicate point of application and assumed direction of unknown applied forces. These usually consist of reactions through which the ground and other bodies oppose the possible motion of the rigid body.
- Include the dimensions necessary to compute the moments of the forces.

From: Rabiei, Chapter 4

Homework Problem 6.1



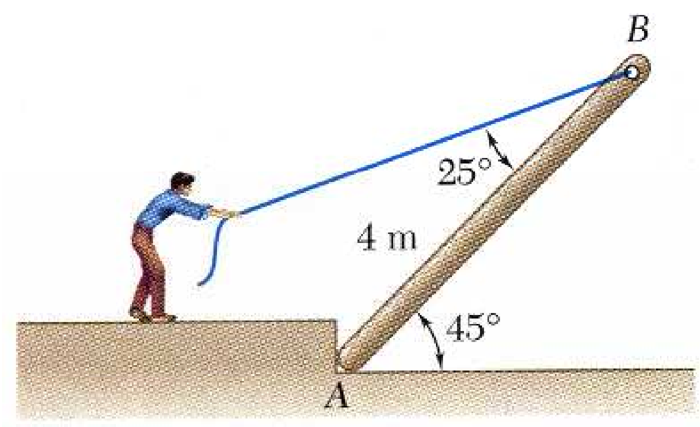
A fixed crane has a mass of 1000 kg and is used to lift a 2400 kg crate. It is held in place by a pin at *A* and a rocker at *B*. The center of gravity of the crane is located at *G*.

Determine the components of the reactions at *A* and *B*.

SOLUTION:

- Create a free-body diagram for the crane.
- Determine *B* by solving the equation for the sum of the moments of all forces about *A*. Note there will be no contribution from the unknown reactions at *A*.
- Determine the reactions at A by solving the equations for the sum of all horizontal force components and all vertical force components.
- Check the values obtained for the reactions by verifying that the sum of the moments about *B* of all forces is zero.

Sample Problem 6.2



A man raises a 10 kg joist, of length 4 m, by pulling on a rope.

Find the tension in the rope and the reaction at *A*.

SOLUTION:

- Create a free-body diagram of the joist. Note that the joist is a 3 force body acted upon by the rope, its weight, and the reaction at *A*.

- The three forces must be concurrent for static equilibrium. Therefore, the reaction ***R*** must pass through the intersection of the lines of action of the weight and rope forces. Determine the direction of the reaction force ***R***.

- Utilize a force triangle to determine the magnitude of the reaction force ***R***.