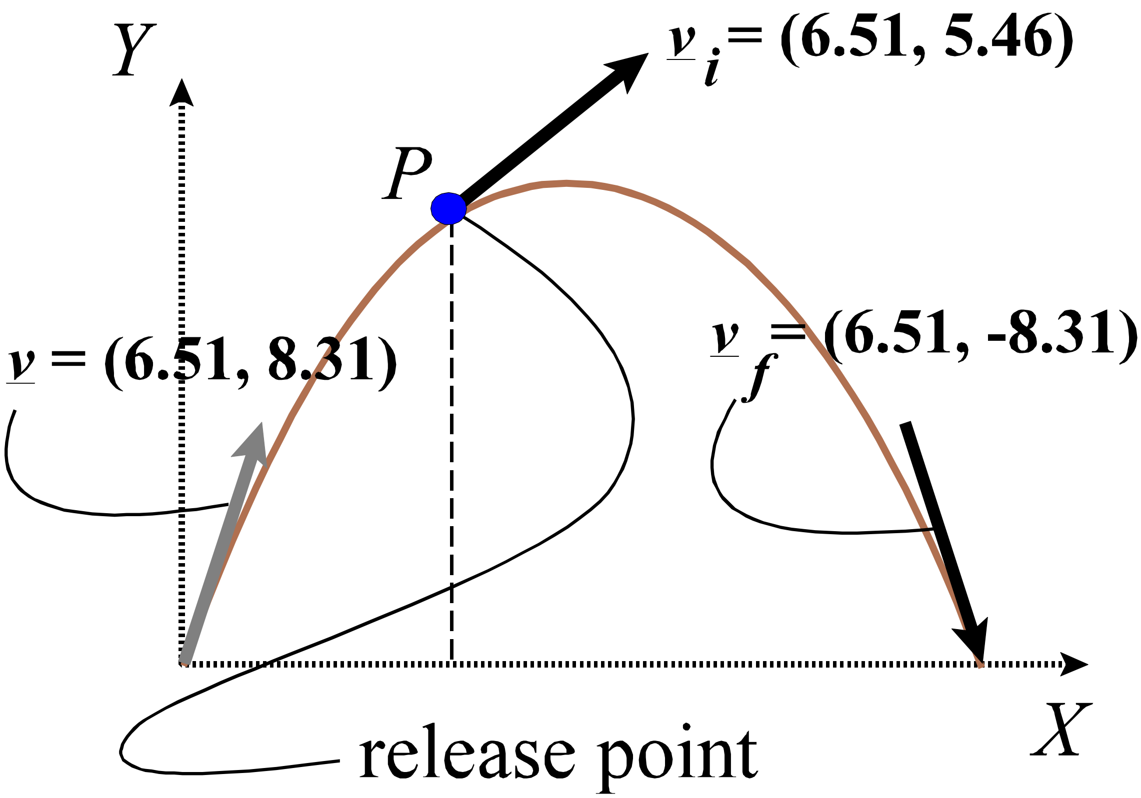
# Trajectory of a Projectile

**Example:**

Ball is released from P (0.0,2.00) with a velocity of (6.51, 5.46) metres per second. Where will it land?




**Solution:**

Cannot find horizontal distance without knowing flight time therefore use vertical motion equations to solve for the time then apply horizontal motion equation to get landing distance.

# Solution

Equation 4 (next page) could be used but you would have to "find the roots of a quadratic equation." This is a difficult problem (unless the initial velocity is zero). An alternate approach is to solve for the final velocity with equation 3, then apply equation 2 to compute the time.

$$
\begin{aligned}
v_{f_y}^{\,2} &= v_{i_y}^{\,2} - 2g(s_{f_y} - s_{i_y}) \\
&= 5.464^2 - 2(9.81)(0 - 2.0) \\
&= 29.86 + 39.24 = 69.1 \\
v_{f_y} &= \pm\sqrt{69.10} = \pm 8.31 \quad [\text{m/s}]
\end{aligned}
$$

Because of the square root operation there are two solutions to this equation. One solution is "extraneous." It represents the velocity "before" the projection started. Thus, we ignore the positive (upwards) velocity and choose the negative velocity as the correct answer.

Now, compute the flight time.

$$
\begin{aligned}
t &= \frac{v_{f_y} - v_{i_y}}{-g} \\
&= \frac{-8.31 - 5.46}{-9.81} = \frac{13.77}{9.81} \\
&= 1.404 \quad [\text{s}]
\end{aligned}
$$

Finally, use the equation for constant speed horizontal motion to compute the landing distance.

$$
\begin{aligned}
s_{f_x} &= s_{i_x} + v_{i_x} t \\
&= 0 + 6.51 \times 1.404 = 9.14 \quad [\text{m}]
\end{aligned}
$$

**Thus, the person will travel 9.14 metres forward.**

# Projectile Motion Equations

$$s_{f_x} = s_{i_x} + v_{i_x} t \qquad (1)$$

$$v_{f_y} = v_{i_y} - gt \qquad (2)$$

$$v_{f_y}^2 = v_{i_y}^2 - 2g(s_{f_y} - s_{i_y}) \qquad (3)$$

$$s_{f_y} = s_{i_y} + v_{i_y} t - 1/2gt^2 \qquad (4)$$

$$s_{f_y} = s_{i_y} + 1/2(v_{i_y} + v_{f_y})t \qquad (5)$$

Where:

$s_i$ = initial position
$s_f$ = final position
$v_i$ = initial velocity
$v_f$ = final velocity
$g$ = acceleration due to gravity = 9.81 $m/s^2$
$t$ = duration in seconds