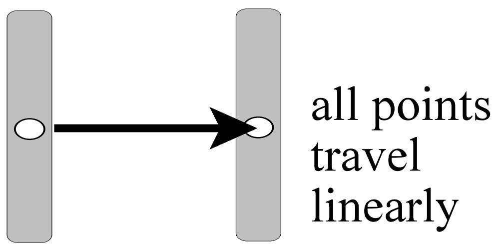
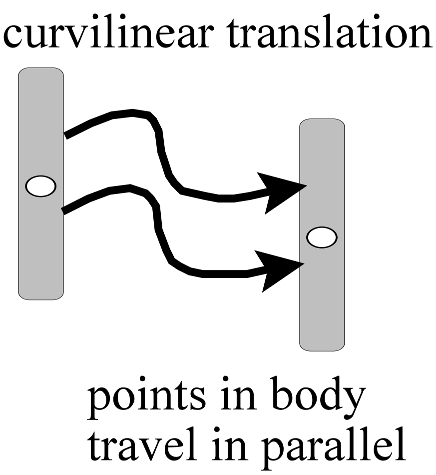
Kinematics

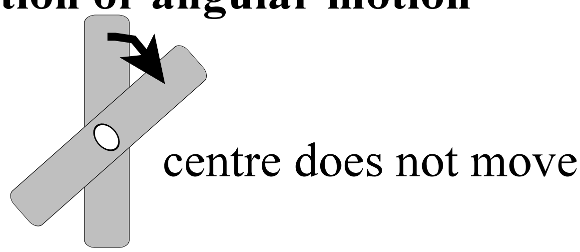
Types of Motion

Rectilinear translation or linear motion

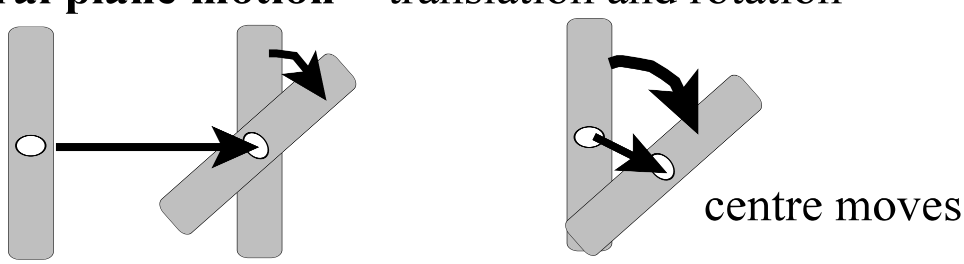




Rotation or angular motion

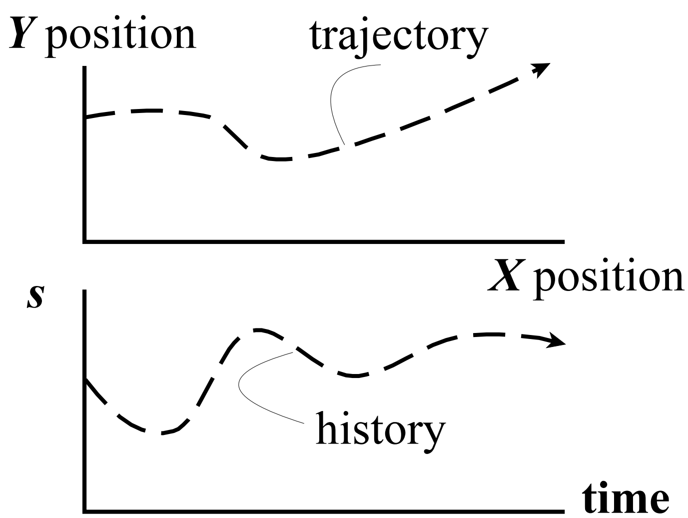


General plane motion = translation and rotation

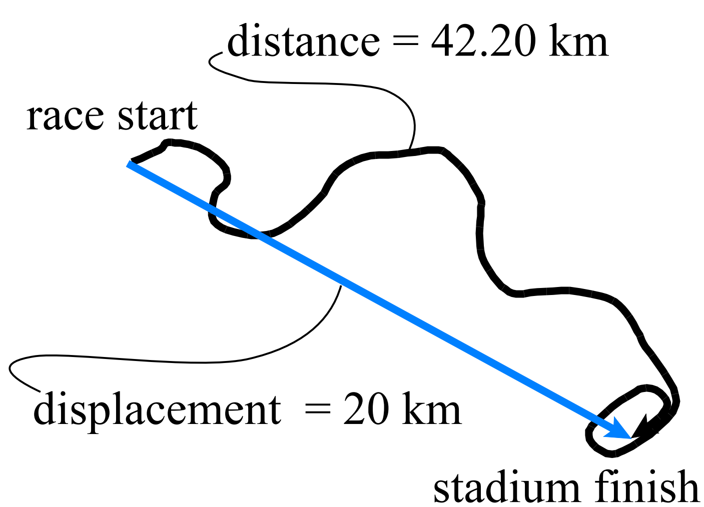


Linear Kinematics

Histories versus Trajectories



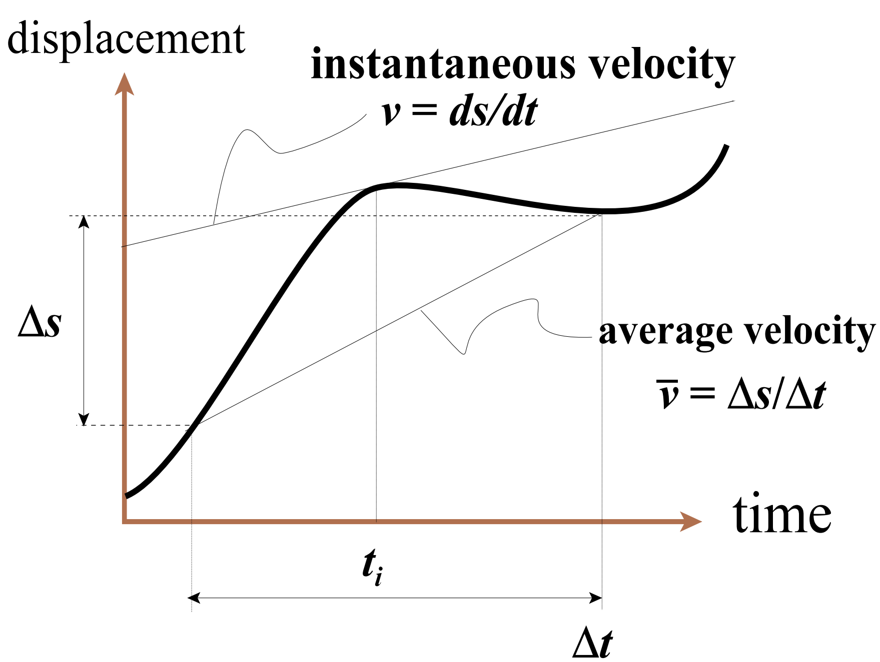
Distance versus Displacement



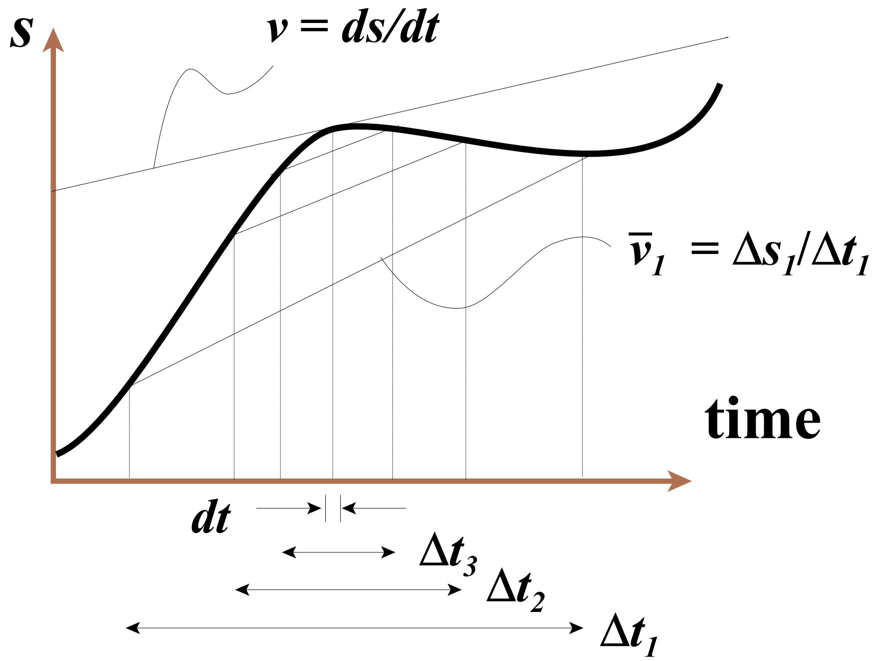
distance = length travelled along a path, a scalar quantity
displacement = vector connecting a point to the origin or from one point to another point later in time (in SI, abbreviation for displacement is ***s***)

Instantaneous Velocity versus Average Velocity

- **speed** is rate of change of distance
- **velocity** is rate of change of displacement
- speed is a scalar quantity, velocity is a vector
- direction of velocity vector is same as direction of motion
- **average velocity** ($\bar{v}$) is the change in displacement over a finite duration, t (delta $\boldsymbol{t}$)
- **instantaneous velocity** ($\boldsymbol{v}$) is slope (tangent) to the displacement-time curve at a particular instant ($\boldsymbol{t_i}$)



Differentiation



instantaneous velocity = $v = \lim_{\Delta t \to 0} \dfrac{\Delta s}{\Delta t} = \dfrac{ds}{dt}$

average velocity = $\bar{v} = \dfrac{\Delta s}{\Delta t}$

Example: What is the average velocity of a person who takes 1.2 seconds to cover a distance of 5.00 m?

$$\bar{v} = \frac{\Delta s}{\Delta t} = \frac{5.00}{1.2} = 4.17 \text{ m/s}$$

 From: Robertson, Chapter 5, Part 1

Acceleration

- rate of change of velocity
- rate of change of rate of change of displacement
- second derivative of displacement with respect to time
- a vector quantity usually in m/s^2
- in Newtons' notation written as: $a = \dot{v} = \ddot{s}$

- in Lebnitz's notation (standard calculus):

instantaneous acceleration $= a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt} = \frac{d^2 s}{dt^2}$

average acceleration $= \bar{a} = \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{\Delta t}$

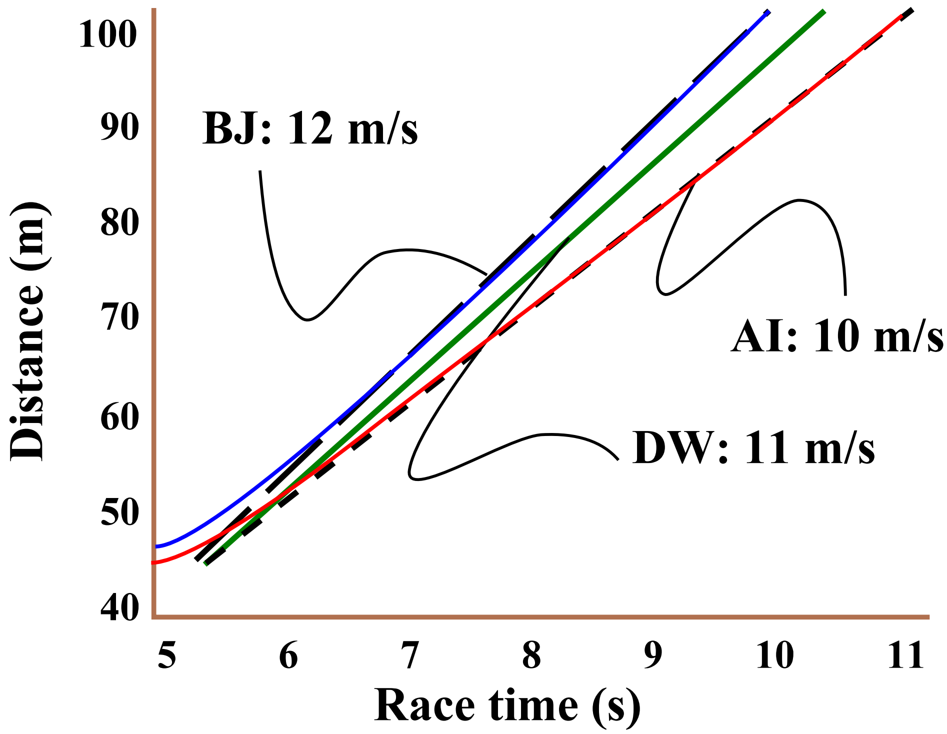
Examples: What is the average acceleration of an object that starts sliding along a tabletop with a velocity of 2.50 m/s and comes to a stop in 3.50 seconds?

$$\bar{a} = \frac{\Delta v}{\Delta t} = \frac{v \; - v_i}{t} = \frac{0 - 2.5}{3.5} = -0.714 \text{ m/s}^2$$

What is the final velocity of a person who accelerates at the rate of 1.5 m/s^2 for 4 seconds from an initial velocity of 2.00 m/s?

$$v_f = v_i + at = 2.00 + 1.5\,(4) = 8.00 \text{ m/s}$$

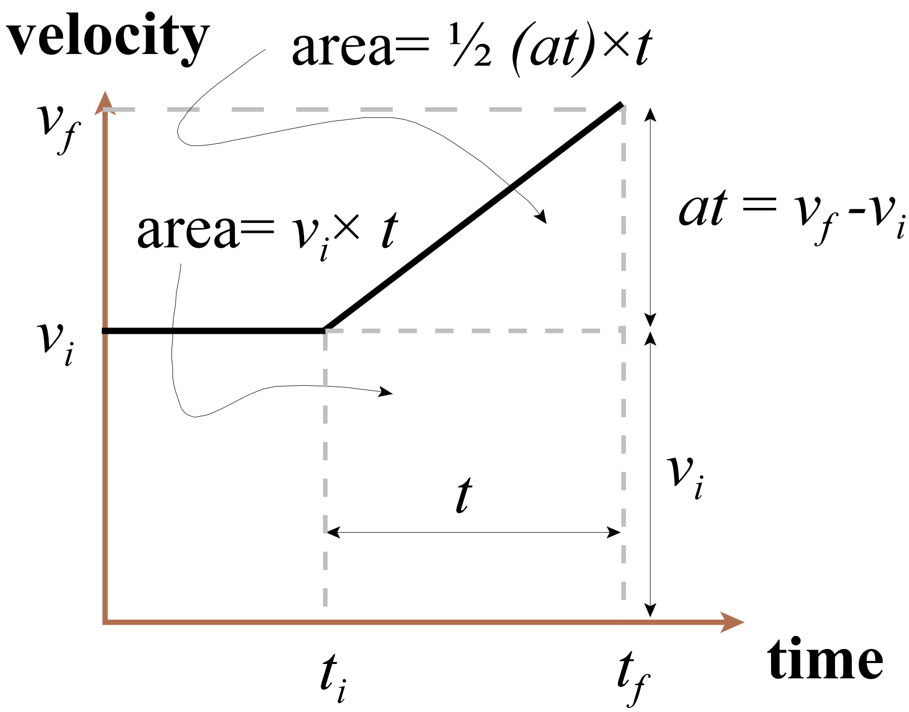
Displacement Histories of Sprinters



- acceleration occurs from start of race until maximum velocity is achieved
- essentially these athletes run last portion of race at their maximum constant velocity
- an athlete may appear to decelerate but may actually be running at a slower velocity

Integration of a Motion with Constant Acceleration

- given a constant velocity of $\boldsymbol{v_i}$ until time $\boldsymbol{t_i}$
- then a constant acceleration, $\boldsymbol{a}$, until time $\boldsymbol{t_f}$
- displacement is area under **velocity** vs. **time** history



Let $\boldsymbol{t = t_f - t_i}$

and $\boldsymbol{a = (v_f - v_i) / t}$

displacement = area of rectangle + triangle

$$\boldsymbol{s_{i\rightarrow f} = v_i\, t + ½\, a\, t^2}$$

Constant Linear Acceleration Equations

$$v_f = v_i + a\,t \qquad \text{(I)}$$
$$s_f = s_i + v_i t + \tfrac{1}{2} a\, t^2 \qquad \text{(II)}$$
$$v_f^{\,2} = v_i^{\,2} + 2a\,(s_f - s_i) \qquad \text{(III)}$$
$$s_f = s_i + \tfrac{1}{2}\,(\,v_i + v_f\,)t \qquad \text{(IV)}$$

Where:

s_i = initial position
s_f = final position
v_i = initial velocity
v_f = final velocity
a = constant linear acceleration
t = duration from initial to final positions (i.e., t)

Equations for Constant Linear Acceleration and Their Relationships

No.	Equation	s_i	v_i	s_f	v_f	t	a
I	$v_f = v_i + at$	✗	✓	✗	✓	✓	✓
II	$s_f = s_i + v_i t + \frac{1}{2}at^2$	✓	✓	✓	✗	✓	✓
III	$v_f^{\,2} = v_i^{\,2} + 2a(s_f - s_i)$	✓	✓	✓	✓	✗	✓
IV	$s_f = s_i + \frac{1}{2}(v_i + v_f)t$	✓	✓	✓	✓	✓	✗