MECHANICAL PROPERTIES OF ARTICULAR CARTILAGE

TENSILE PROPERTIES

The tensile behavior of articular cartilage (AC) qualitatively is much like the other passive soft tissues (ligament & tendon) and is dependent on the collagen content and organization. In fact, similar nonlinear constitutive laws for the tensile pseudoelastic response are used, as in the familiar

$$\sigma_e(\lambda) = \beta\left(e^{\alpha\lambda} - 1\right)$$

where T is the Lagrange stress, λ the stretch ratio, and α and β constants. An instantaneous modulus E can be found by differentiating this relation with respect to λ, to give

$$E = \frac{d\sigma_e}{d\lambda} = \alpha\left(\sigma_e + \beta\right)$$

Tension stresses can be developed in AC in a manner akin to "hoop tension," as material near a contact regions resists expansion. A physiologically reasonable estimate of tensile stress that can be developed *in vivo* is up to 2.5 MPa. In any event, tensile tests are fairly easy to perform, as was done by Weightman in 1976 and later by Woo et al. Weightman found cartilage specimens to have static strengths of about 20 MPa and infinite fatigue lives up to stress of 10 MPa. Woo et al. investigated still smaller specimens on a zone by zone basis. They found that cartilage was stiffer in the superficial tangential zone in the direction of collagen fibers (as opposed to the transverse direction). They further found stiffness to decrease moving deeper into AC down to the deep zone.

FRICTION

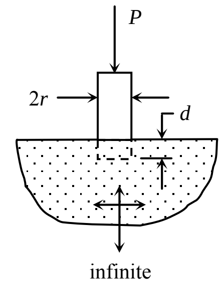
The coefficient of static friction μ_s for synovial joints has been measured to be less than 0.01, compared to aluminum on aluminum value of about 2. Thus, for a 700 N person (and just considering body weight to act on the knee joints), a anteroposterior (or any other translational direction) shear force of only 3.5 N (less than a pound) applied to the femur is necessary to begin moving it relative to the tibia. Note that friction is rate dependent. Preconditioning the cartilage (as in stretching and warming up in athletic activities) has been shown to cause less strain in cartilage for same applied loads.

INDENTATION PROPERTIES

Preparing tiny specimens of (articular cartilage) AC for mechanical testing can be problematic. In addition, since loading of AC *in vivo* is predominantly through compressive contact between articular surfaces, a reasonable method of testing that can achieve this type of loading is indentation testing. This method can simultaneously distinguish heterogeneity in mechanical properties, provided the indentor is small enough. Analyses of the mechanics of indentation fall under the broad class of contact problems, which are characterized by their nonlinear load-deformation behavior. In all of the following analyses, AC was assumed to be isotropic and homogeneous and the indentor to be cylindrical of radius r with a flat end (contrary to what is reported in the Martin et al. chapter). An elastic modulus E was then computed from the analytical expressions and experimental data.

Sokoloff (1966) developed a stress analysis for the indentation of a rigid and incompressible half space. A schematic of the problem analyzed is given in the figure at right, in which the applied load is P and the indentation depth is d. The resulting expression for the modulus was found to be

$$E = \frac{P\left(1-\nu^2\right)}{2rd}$$

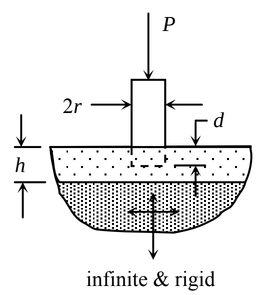


An incompressible material is a material that undergoes no volume change when stressed. For example, consider the uniaxial stress state such that the only nonzero stress component is $\sigma_1 = \sigma$. The dilatation e by definition is the sum of the three normal strains, or, here,

$$e = \varepsilon_1 + \varepsilon_2 + \varepsilon_3 = \frac{\sigma}{E}\left(1-2\nu\right)$$

For an incompressible isotropic material then, the Poisson's ratio must be ½. With these assumptions, the initial modulus (during the first load application after no preconditioning) of AC was found to be about 2.3 MPa. Kempson et al (*Journal of Biomechanics* 1971) later applied a correction factor to the Sokoloff solution to account for the AC thickness to determine an initial modulus of 2.25 MPa and an aggregate modulus of 0.69 MPa. The aggregate modulus can be thought of as the steady state or equilibrium modulus under a specified load after all fluid movement has ceased.

Hayes (*Journal of Biomechanics* 1972) supplied the solution to a thin elastic layer of thickness h supported by a rigid half space that included a scale factor κ to account for the deformation and Poisson's ratio of the elastic layer and the diameter of the indentor. A schematic of this problem is given in the figure at right. The resulting expression for the modulus was found to be



$$E = \frac{P\left(1-\nu^2\right)}{2rd\kappa\left(\frac{r}{h},\nu\right)}$$

An initial modulus of 1.68 MPa and an aggregate modulus of 0.56 MPa was then determined when a Poisson's ratio of 0.4 was assumed for the AC.

BIPHASIC THEORY

Background

When articular cartilage (AC) is loaded, for example, when a cadaveric tibial plateau is depressed with a blunt instrument, drops of fluid can be observed to appear on the surface. Clearly, fluid is moving within and out of the tissue, contributing to its mechanical (and viscoelastic) behavior. Many researchers have conducted controlled tests to determine the interaction between AC deformation and fluid movement, which eventually led to the development by Mow and associates of the biphasic theory of AC.

Basic Assumptions

The basic assumptions behind the biphasic theory are as follows. AC is assumed to consist of two incompressible phases: (1) a soft porous solid phase (to represent the collagen) whose pores are filled with (2) a fluid phase (to represent the water). The pores are assumed to be interconnected so that the fluid flows throughout the tissue with resistance offered by the intrinsic permeability of the tissue. No closed, fluid filled pores exist to bolster the ability of the solid phase to support load. Any observed volume change of the tissue results from fluid exudation or imbibition.

Permeability

The hydraulic permeability coefficient (*aka* apparent permeability or simply permeability), denoted by k with SI units of $\left[\dfrac{\mathrm{m}^4}{\mathrm{N}\cdot\mathrm{s}}\right]$, is a measure of the resistance to fluid movement offered by a porous material. It represents the distance a unit volume of fluid moves through a unit area of material under the action of a unit pressure in one unit of time. Thus, in SI units, a porous material with a permeability of $k = 1\dfrac{\mathrm{m}^4}{\mathrm{N}\cdot\mathrm{s}}$ is interpreted as a material that allows 1 m^3 of fluid to move 1 m in 1 s through an area of 1 m^2 under the action of a pressure of 1 Pa. Clearly a length scale based on meters for volume, area, and distance are large for AC, so it is not expected for permeabilities in SI units to be on the order of whole numbers. In fact, they are quite small. For AC (and many materials), permeability is a function of deformation, so that as the tissue compacts the permeability decreases (eventually becoming impermeable, for which $k = 0$).

Important Tests

Although the development of the biphasic theory is beyond the scope of our course, it is important to consider the solutions offered by it for mechanical tests traditionally performed for constitutive model development.

Confined Compression Tests. In confined compression tests of AC, a cylindrical specimen of initial height h is placed in a cylinder with solid metal walls and a solid bottom (Figure 1). Keep in mind that the initial height of these types of specimens are at most a few millimeters. A highly porous, disk shaped platen of cross sectional area A is used to supply the compression (like a piston), in the form of an applied load $F(t)$ or an applied deformation $u(t)$; if one is applied, the other is measured. A stress may be defined as $\sigma(\tau) = F(t)/A$ and a strain $\varepsilon = u(t)/h$. The resulting load-deformation data are fit with the solutions provided by the biphasic theory to estimate the material constants k and H_A.

Confined Compression Creep Test. In the creep version of the confined compression tests, the usual step function load $F(t) = F_0H(t)$ is applied to a specimen so that $\sigma(t) = \sigma_0 H(t)$ where $F_0 = \sigma_0 A$, and the resulting deformation $u(t)$ is recorded. The load-deformation solution provided by the biphasic theory is

$$\frac{u(t)}{h} = \frac{\sigma_0}{H_A}\left\langle 1 - \frac{2}{\pi^2}\sum_{n=0}^{\infty}\left\{\frac{\exp\left[-\dfrac{(n+½)^2\pi^2 H_A k}{h^2}t\right]}{(n+½)^2}\right\}\right\rangle$$

Note that initially ($t = 0$)

$$\frac{u(t=0)}{h} = \frac{\sigma_0}{H_A}\left\langle 1 - \frac{2}{\pi^2}\sum_{n=0}^{\infty}\left\{\frac{1}{(n+½)^2}\right\}\right\rangle = \frac{\sigma_0}{H_A}\left\langle 1 - \frac{2}{\pi^2}\frac{\pi^2}{2}\right\rangle = 0$$

because of the convergence of the infinite series. Also note that for the steady state condition ($t \to \infty$)

$$\frac{u(t\to\infty)}{h} = \frac{\sigma_0}{H_A}\left\langle 1 - \frac{2}{\pi^2}\sum_{n=0}^{\infty}\left\{\frac{0}{(n+½)^2}\right\}\right\rangle = \frac{\sigma_0}{H_A}\langle 1-0\rangle = \frac{\sigma_0}{H_A}$$

which is a Hooke's Law type relation (in compliance form) between strain and stress.

Confined Compression Relaxation Test. Similar relations are provided by the biphasic theory for the relaxation version of the confined compression tests. The solution presented here represents the manner in which an actual relaxation test is performed: by ramping up the deformation $u(t)$ over a finite period of time from $t = 0$ to $t = t_0$ at a rate of $\dot{u}$, then holding the deformation constant and recording the load relaxation $F(t)$. The resulting load-deformation solution provided by the biphasic theory and valid over $t \geq t_0$ is

$$\frac{\sigma(t \geq t_0)}{H_A} = -\frac{\dot{u}}{h}\left\{t_0 - 2\sum_{n=1}^{\infty}\left[\frac{\exp(-Kt)}{K} - 1\right]\right\} \quad \text{where} \quad K = \frac{n^2\pi^2 H_A k}{h^2}$$

Note that for the steady state condition ($t \to \infty$)

$$\frac{\sigma(t\to\infty)}{H_A} = \frac{\dot{u}\,t_0}{h} = \frac{u(t=t_0)}{h}$$

which is a Hooke's Law type relation (in stiffness form) between stress and strain. The stress response (Figure 2) is typical for a relaxation test, and reflects the fluid movement and solid compaction that occurs.

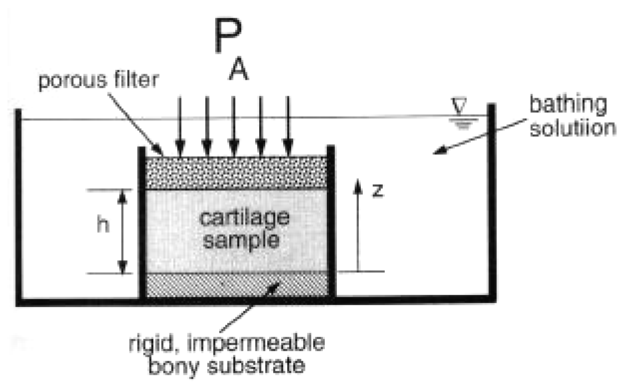


Figure 1. Schematic of confined compression test set up. The sample can not extend laterally due to the rigid test chamber.

From Mow & Hayes *Basic Orthopaedic Biomechanics* 1997

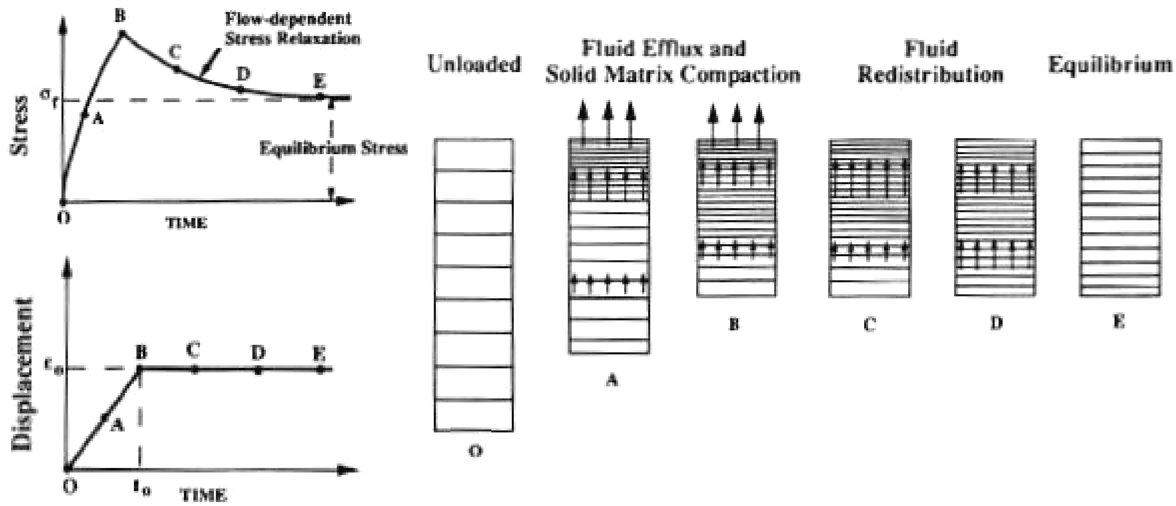


Figure 2. Schematic of confined compression relaxation test data. At bottom left, a plot representing a realistic displacement input, ramping to a constant compression in a finite time t_0. At left top left, a plot representing the stress response. At right, schematics of the articular cartilage test specimen at various times during the test. The horizontal bars are meant to indicate strain within the tissue. Initially (point 0), the tissue is unloaded. During application of the compressive displacement (points A & B), fluid is flowing up through the tissue and through the porous platen; the solid matrix is compacting. The stress response in the phase is a "softening" response, i.e., the tissue is becoming less stiff because the fluid exudation means there is less fluid to resist the applied load. As the displacement is held constant (points C & D), the fluid internally redistributes, until equilibrium is reached (point E and beyond).

From Mow & Hayes *Basic Orthopaedic Biomechanics* 1997