VISCOELASTICITY

INTRODUCTION

- When real materials are stretched, rearrangements within a real material take place over a finite length of time in response to stretch.

- These rearrangements can take place:
  - nearly instantaneously, as when water is made to flow, or
  - seemingly forever, as when steel is slightly stretched at room temperature.

- We observe the material response over a finite length of time.

- A dimensionless measure (after Marcus Reiner, 1886 - 1976, Austrian born Israeli civil engineer) of the response is the Deborah number, $N_D = \dfrac{\text{material response time}}{\text{observation time period}}$, such that:
  - $N_D \to 0$ for a purely viscous response
  - $N_D \to \infty$ for a purely elastic response
  - $N_D \sim 1$ for a viscoelastic response.

- Viscoelastic deformations are characterized by their
  - time dependence
  - loading history dependence
  - lack of one to one correspondence between stress and strain.

- We will first learn about linear viscoelasticity applicable to infinitesimal deformations.

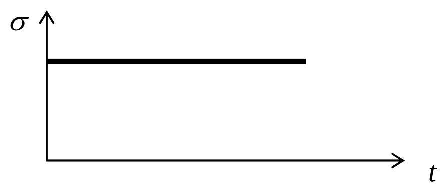
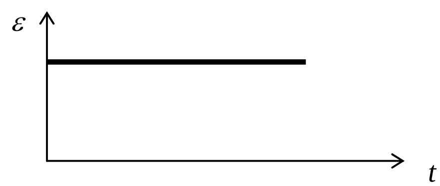
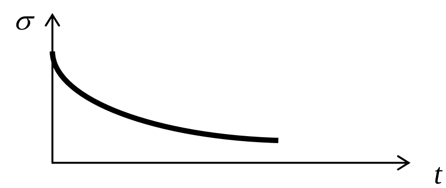
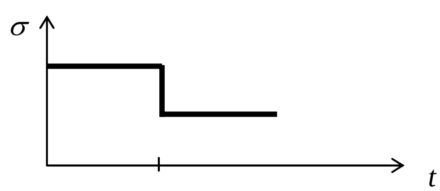
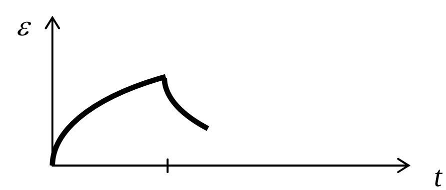
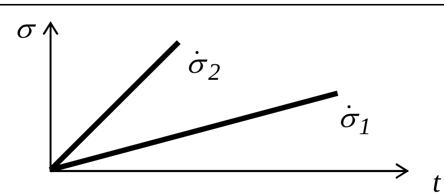
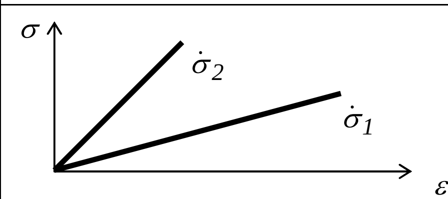
- Later, we will consider finite deformations of soft tissues within the framework of a "quasilinear" viscoelasticity theory.

Observed Viscoelastic Characteristics

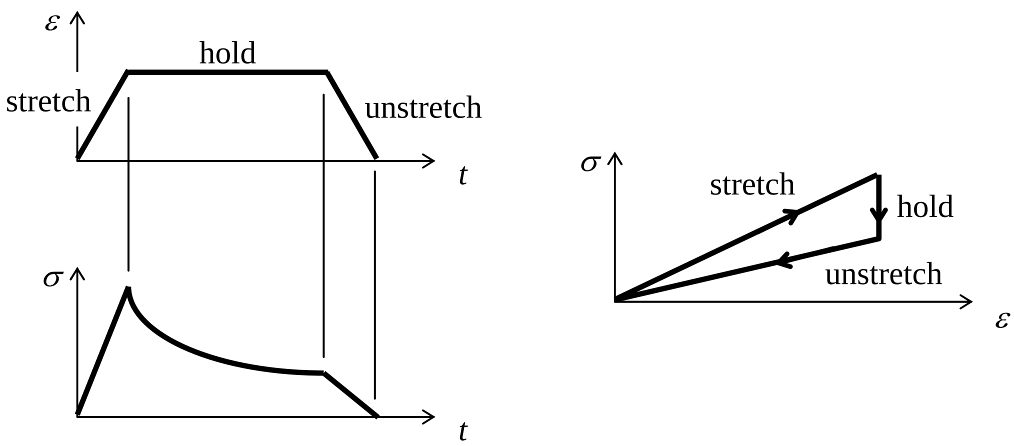
- Unless otherwise noted, our focus will be on uniaxial extension.

- We must master the terminology of viscoelasticity.

- Simple models will help us master idealized responses.

Characteristic	Excitation	Response
Creep	constant stress (load)	increasing strain (deformation)
		
Relaxation	constant strain	decreasing stress
		
Recovery	decrease stress	decreased strain
		
Rate Effects	increase stress (or strain) rate	increased instantaneous stiffness
		

- Another observed characteristic is hysteresis: loading and unloading curves not identical



Linear Viscoelasticity

- Perform (infinitely) many relaxation tests at various values of constant strain $(\varepsilon_0)_i$
  - Mathematically represented by $\varepsilon_i(t) = \mathrm{H}(t)(\varepsilon_0)_i$ where $\mathrm{H}(t)$ is the Heaviside step function
  - Define relaxation modulus as $E_i(t) \equiv \dfrac{\sigma_i(t)}{(\varepsilon_o)_i}$

- Material is linearly viscoelastic if $E_1(t) = E_2(t) = \cdots = E_i(t) = E(t)$

- An analogous linear viscoelastic creep compliance exists such that $J(t) \equiv \dfrac{\varepsilon(t)}{\sigma_0}$