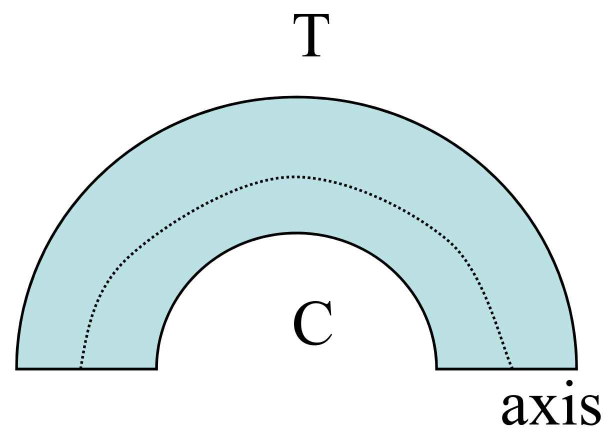
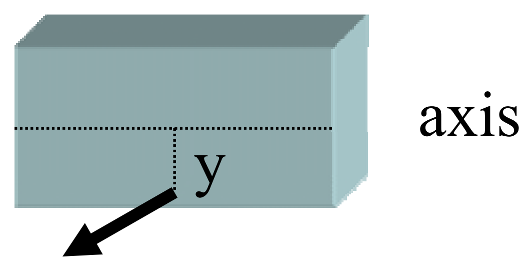
Section 15: Introduction to Stress and Bending

Bending

- Long bones: beams
- Compressive stress: inner portion
- Tensile stress: outer portion
- Max stresses near the edges, less near the neutral axis



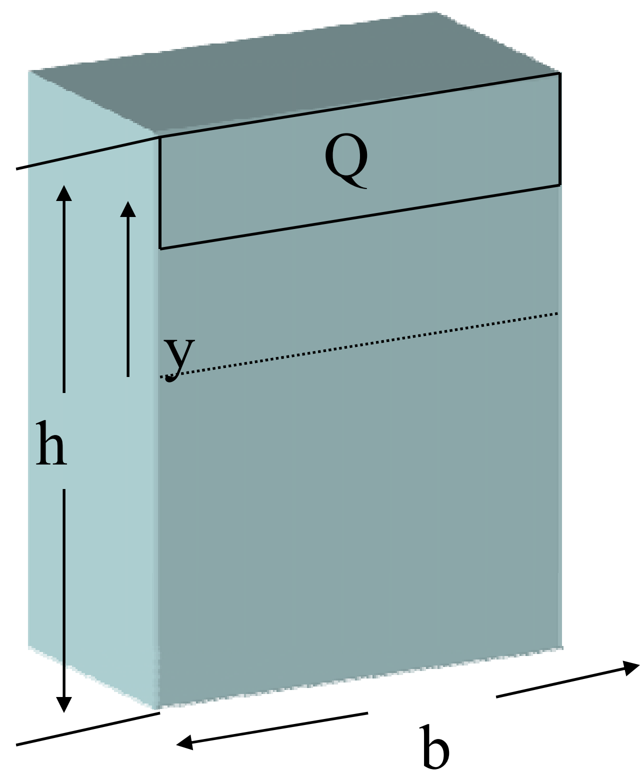


$$\sigma_x=(M_b \cdot y)/I$$

From: Noffal

Bending Moments

- Shear stresses max at neutral axis and zero at the surface
- $\tau = (Q \cdot V)/(I \cdot b)$
- Q= area moment
- V= vertical shear force



From: Noffal

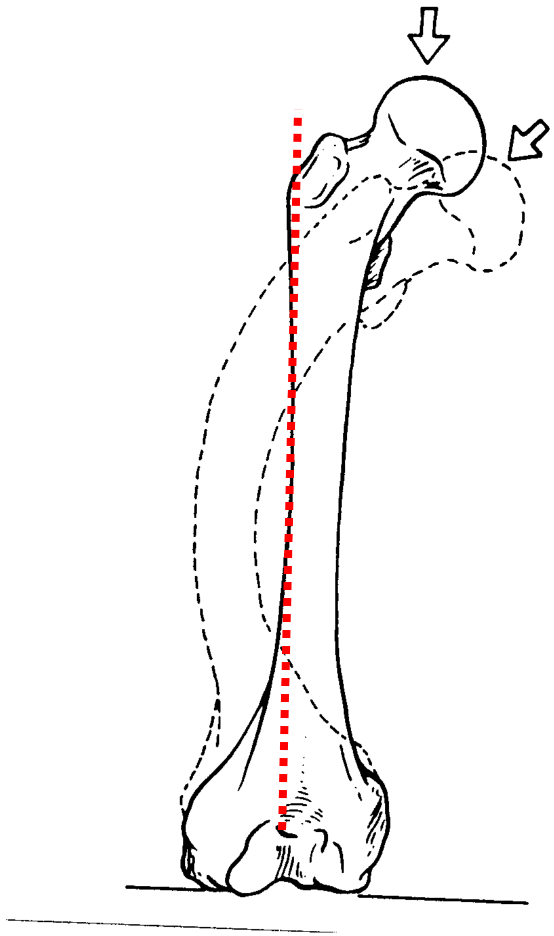
Behavior of Bone Under Bending

- Bending subjects bone to a combination of tension and compression (tension on one side of neutral axis, compression on the other side, and no stress or strain along the neutral axis)
- Magnitude of stresses is proportional to the distance from the neutral axis (see figure)
- Long bone subject to increased risk of bending fractures

From: Brown

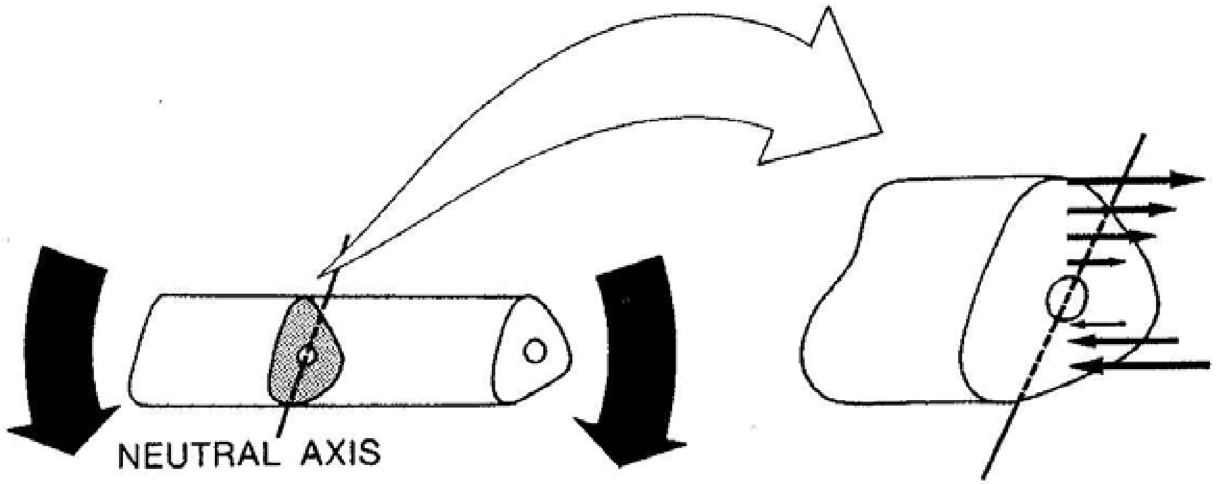
Bending

- Cantilever bending
- Compressive force acting off-center from long axis



From: Noffal

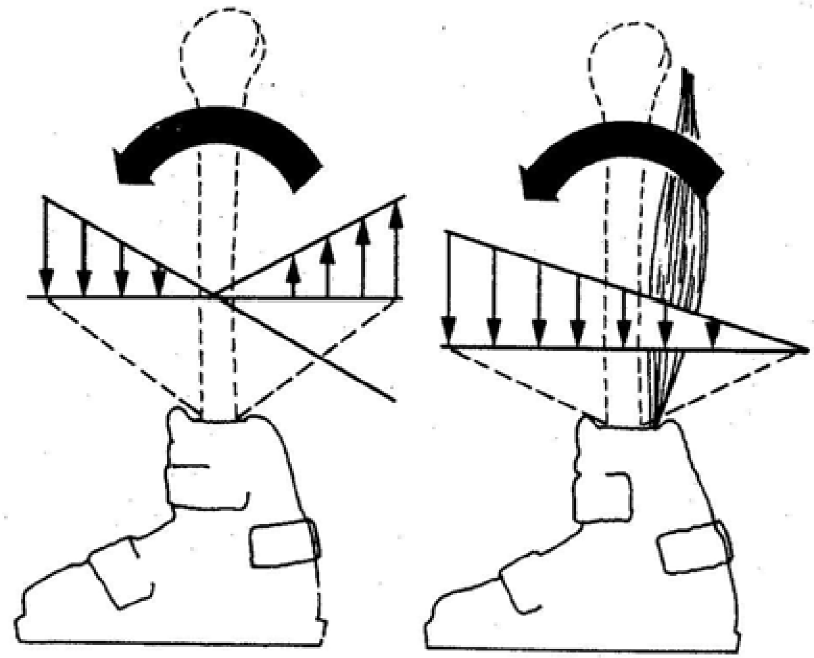
Bending Loading



Cross section of a bone subjected to bending, showing distribution of stresses around the neutral axis. Tensile stresses act on the superior side, and compressive stresses act on the inferior side. The stresses are highest at the periphery of the bone and lowest near the neutral axis. The tensile and compressive stresses are unequal because the bone is asymmetrical.

From: Brown

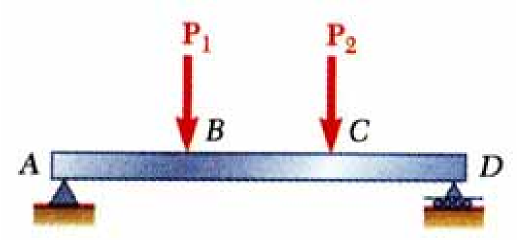
Muscle Activity Changing Stress Distribution



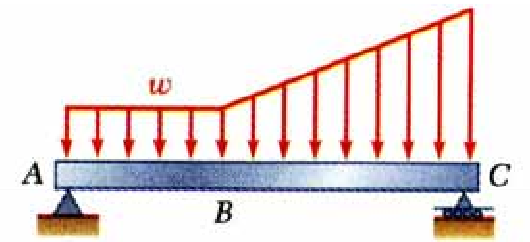
A. Distribution of compressive and tensile stresses in a tibia subjected to three-point bending. **B.** Contraction of the triceps surae muscle produces high compressive stress on the posterior aspect, neutralizing the high tensile stress.

From: Brown

Various Types of Beam Loading and Support



(a) Concentrated loads



(b) Distributed load

- *Beam* - structural member designed to support loads applied at various points along its length.
- Beam can be subjected to *concentrated* loads or *distributed* loads or combination of both.
- *Beam design* is two-step process:
  1) determine shearing forces and bending moments produced by applied loads
  2) select cross-section best suited to resist shearing forces and bending moments

From: Rabiei

- In order to design a beam, it is necessary to determine the maximum shear and moment in the beam
- Express **V** and **M** as functions of arbitrary position *x* along axis.
- These functions can be represented by graphs called *shear and moment diagrams*
- Engineers need to know the *variation* of shear and moment along the beam to know where to reinforce it

Diagrams

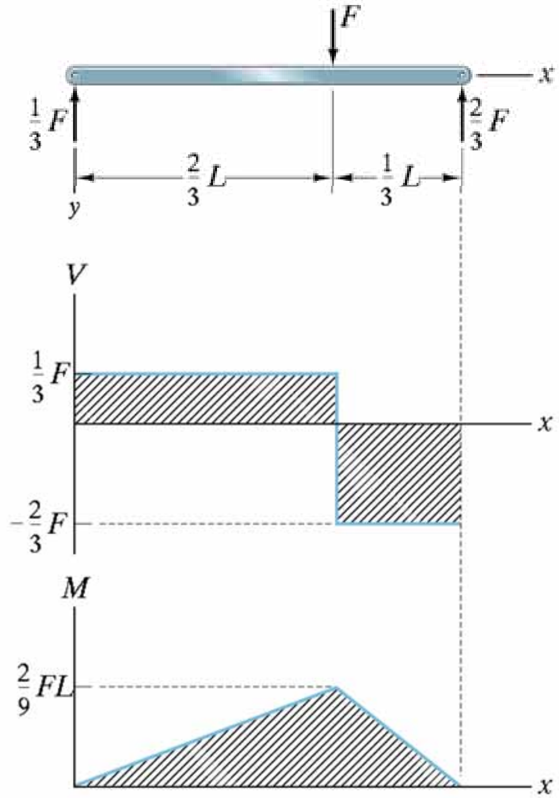
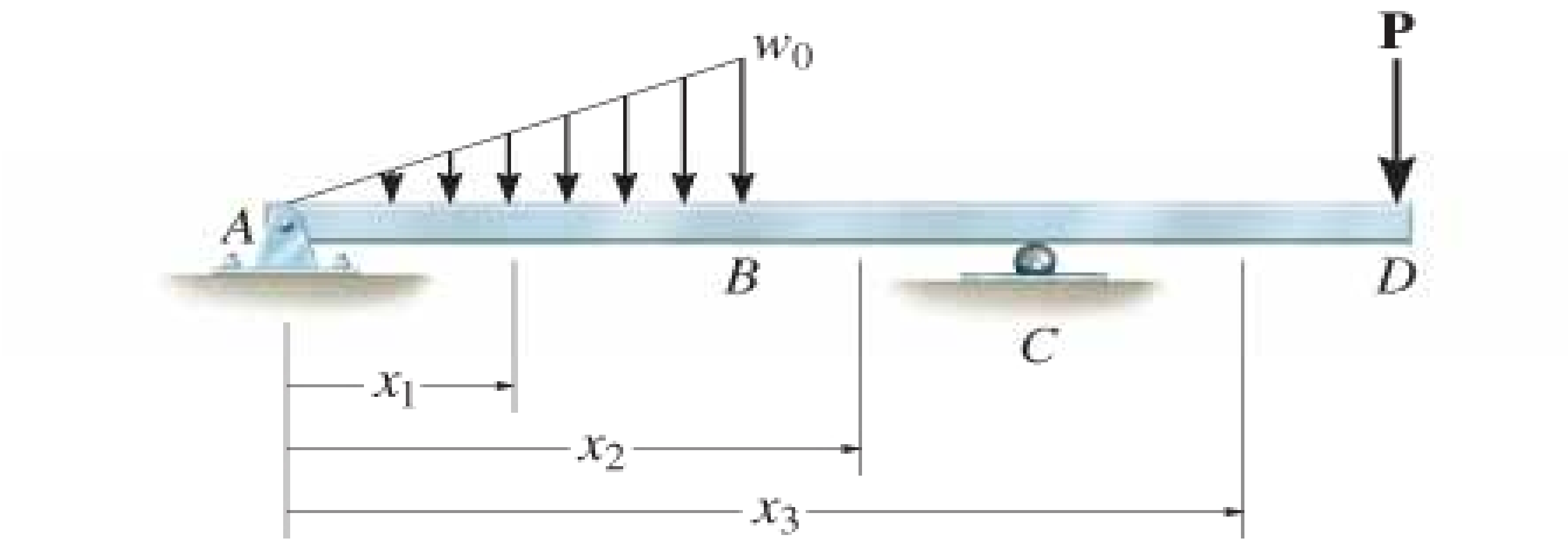


FIGURE 7-7 Shear force and bending moment diagrams indicating the maximum positive and negative values of V and M.

Bedford/Liechti, Mechanics of Materials, 1e, ©2001, Prentice Hall

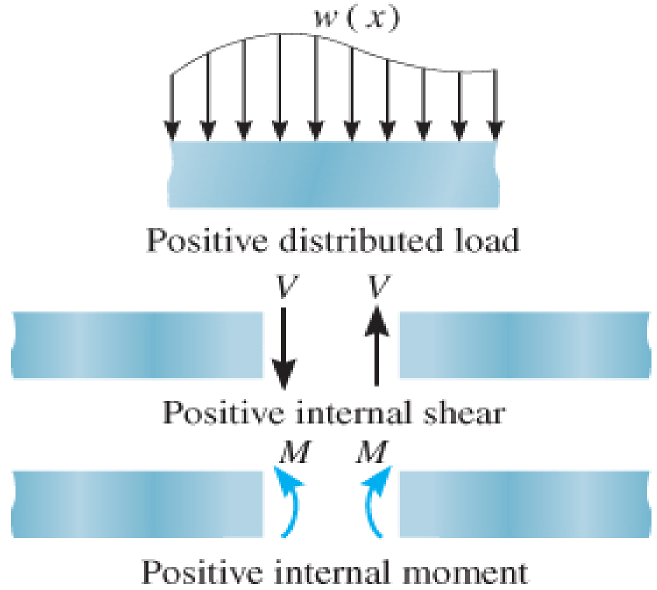
- Shear and bending-moment functions must be determined for each *region* of the beam *between* any two discontinuities of loading



From: Wang

Beam sign convention

- Although choice of sign convention is arbitrary, in this course, we adopt the one often used by engineers:



Beam sign convention

From: Wang

Procedure for analysis

Support reactions

- Determine all reactive forces and couple moments acting on beam
- Resolve all forces into components acting perpendicular and parallel to beam's axis

Shear and moment functions

- Specify separate coordinates x having an origin at beam's left end, and extending to regions of beam between concentrated forces and/or couple moments, or where there is no discontinuity of distributed loading

Procedure for analysis

Shear and moment functions

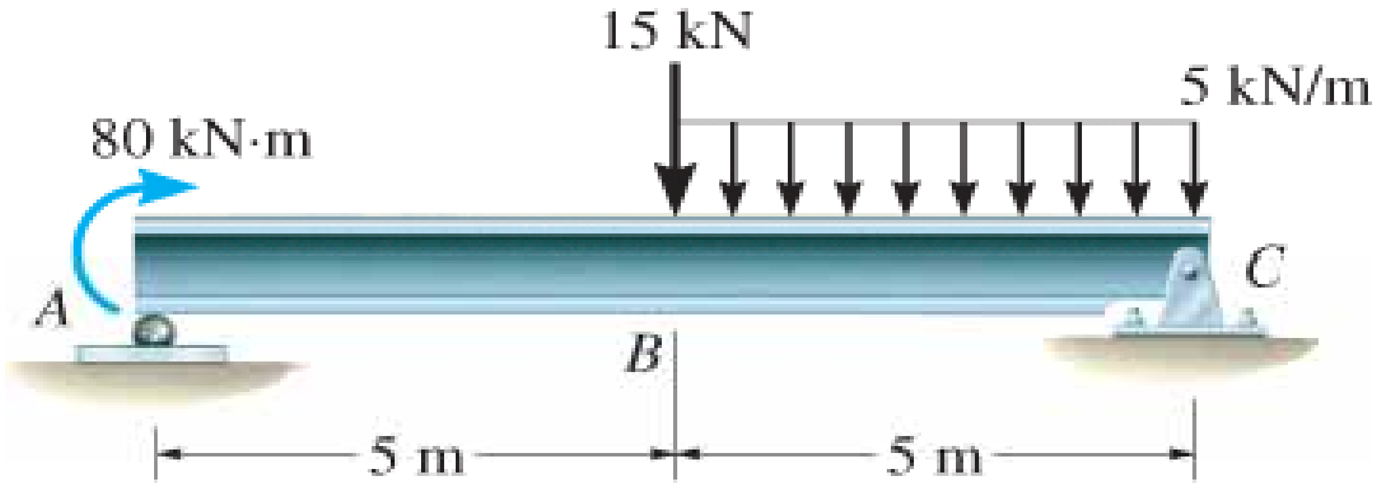
- Section beam perpendicular to its axis at each distance *x*
- Draw free-body diagram of one segment
- Make sure **V** and **M** are shown acting in positive sense, according to sign convention
- Sum forces perpendicular to beam’s axis to get shear
- Sum moments about the sectioned end of segment to get moment

Procedure for analysis

Shear and moment diagrams

- Plot shear diagram (**V** vs. *x*) and moment diagram (**M** vs. *x*)
- If numerical values are positive, values are plotted above axis, otherwise, negative values are plotted below axis
- It is convenient to show the shear and moment diagrams directly below the free-body diagram

Draw the shear and moment diagrams for beam shown below.



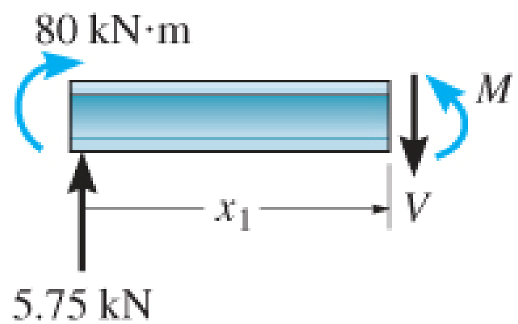
(a)

From: Wang

Support reactions: Shown in free-body diagram.

Shear and moment functions

Since there is a discontinuity of distributed load and a concentrated load at beam's center, two regions of *x* must be considered.



(b)

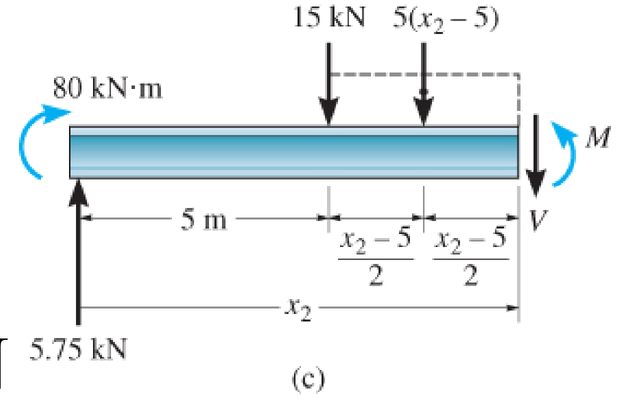
$$0 \le x_1 \le 5 \text{ m},$$

$$+\uparrow \Sigma F_y = 0; \quad \ldots \qquad V = 5.75 \text{ N}$$

$$\circlearrowleft + \Sigma M = 0; \quad \ldots \qquad M = (5.75x_1 + 80) \text{ kN·m}$$

From: Wang

Shear and moment functions



$5\text{ m} \le x_2 \le 10\text{ m},$

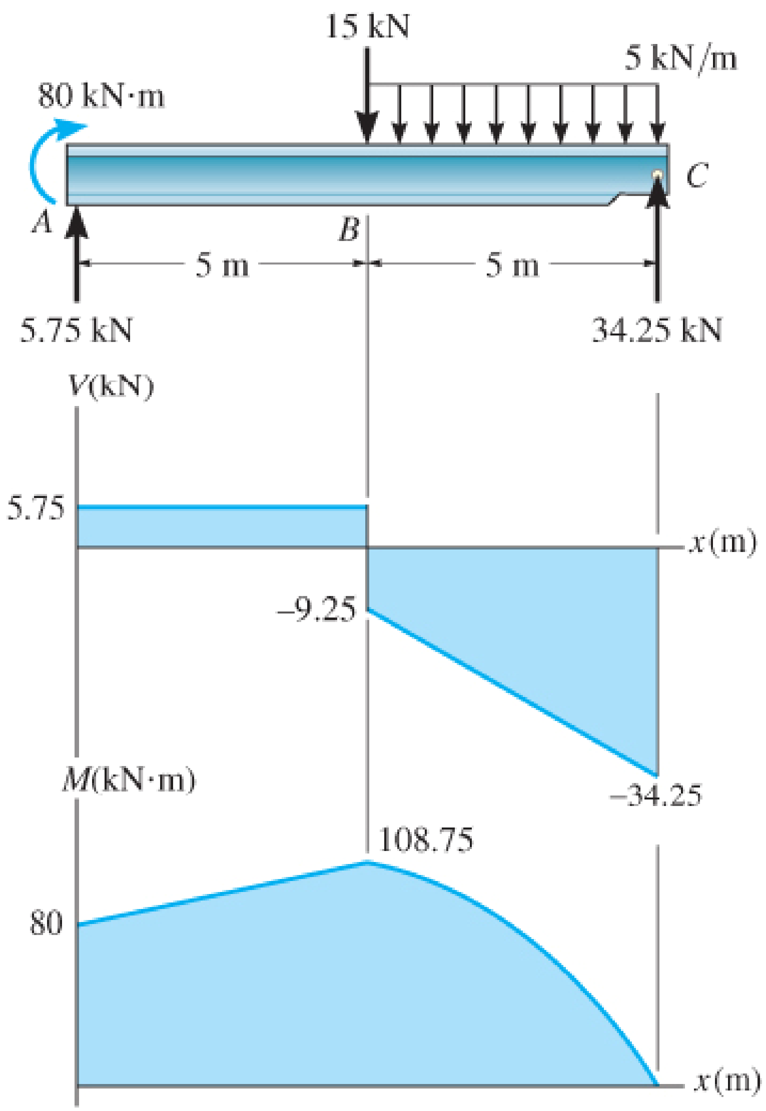
$+\uparrow \Sigma F_y = 0;$ ... $V = (15.75 - 5x_2)\text{ kN}$

$+ \Sigma M = 0;$ ... $M = (-5.75x_2^2 + 15.75x_2 + 92.5)\text{ kN·m}$

Check results by applying $w = dV/dx$ and $V = dM/dx$.

From: Wang

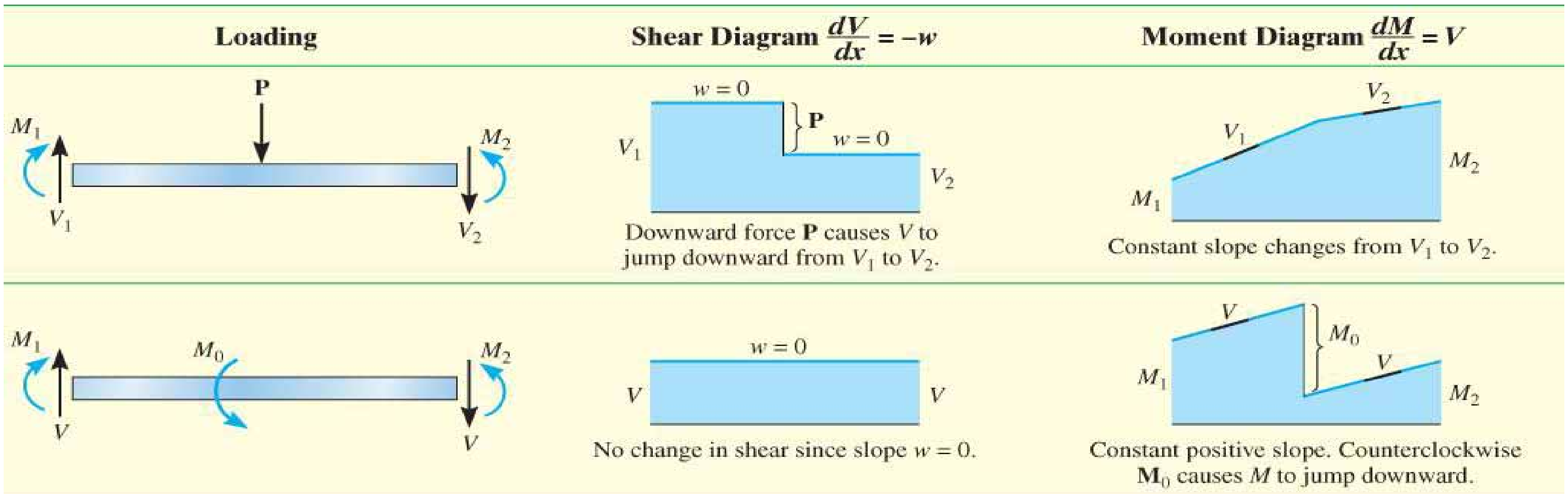
Shear and moment diagrams



(d)

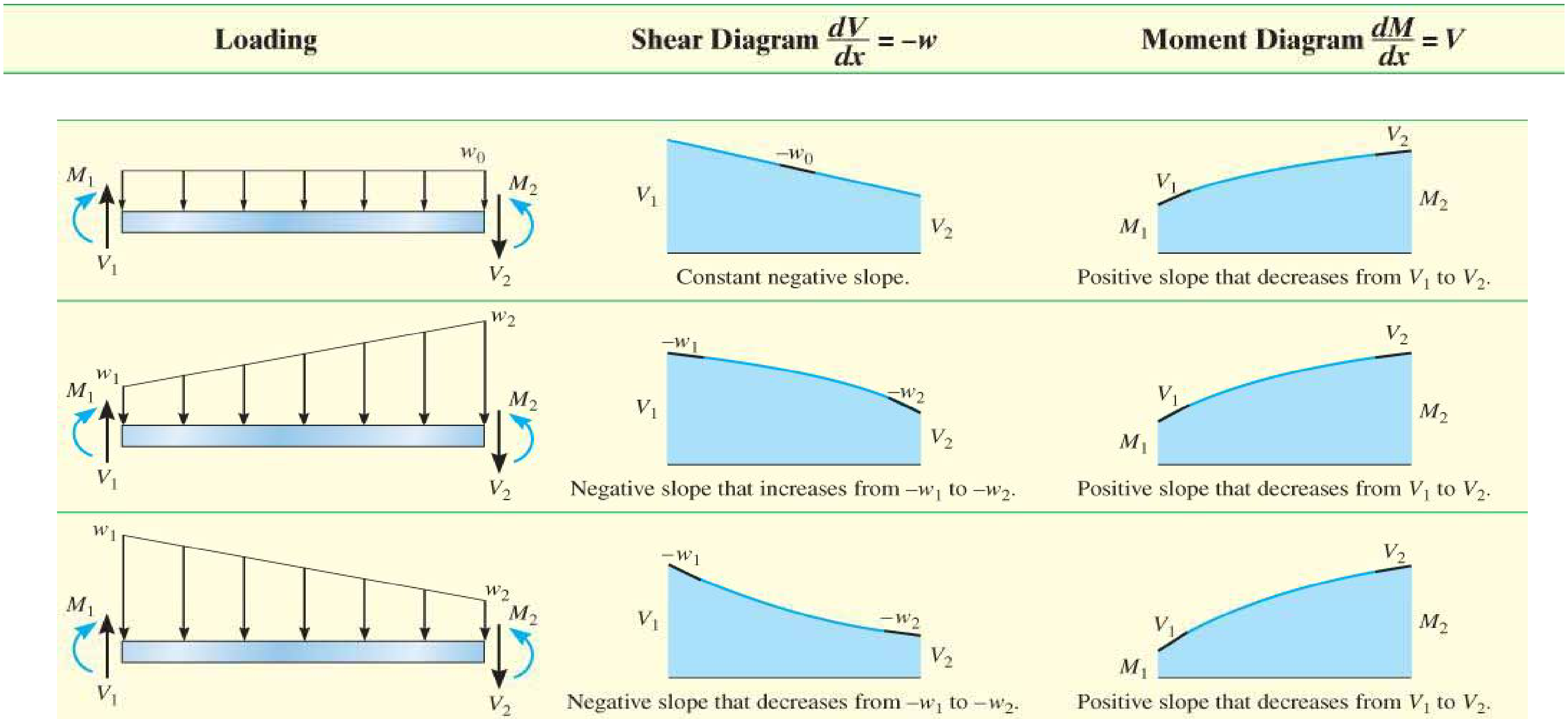
From: Wang

Regions of concentrated force and moment



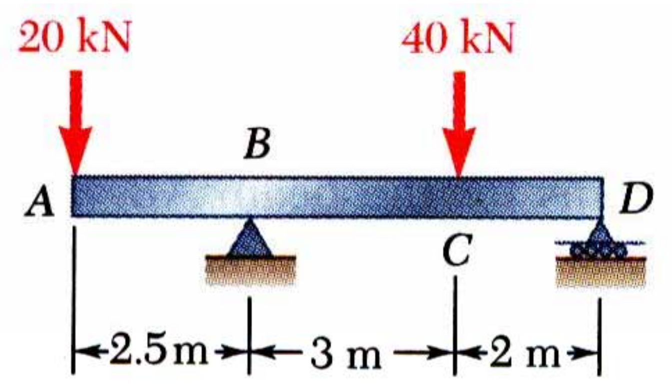
From: Wang

Regions of concentrated force and moment



From: Wang

Sample Problem 7.2



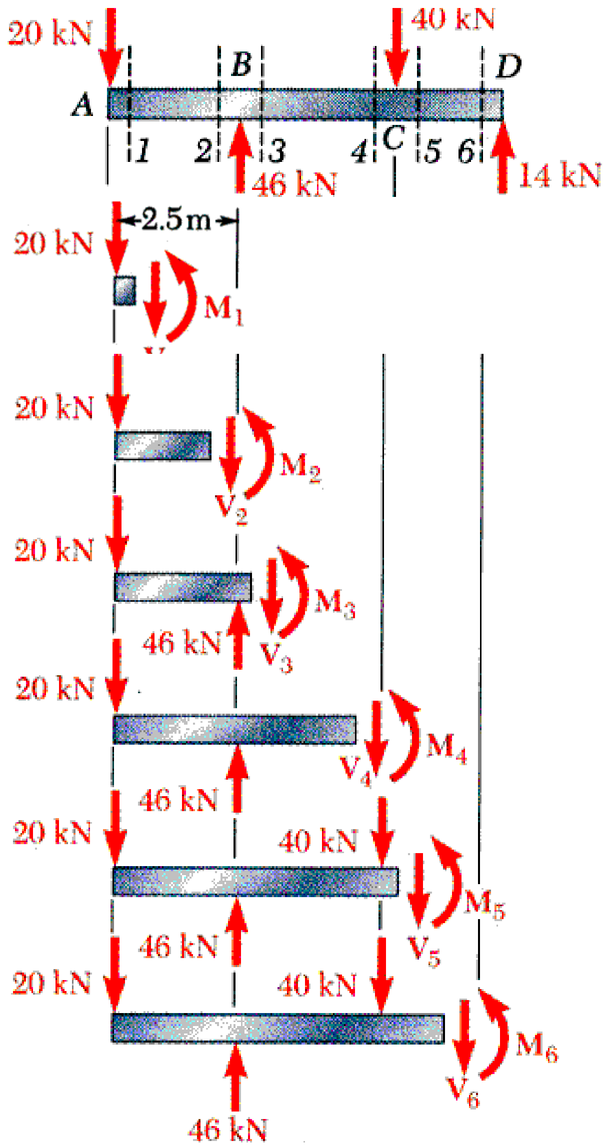
Draw the shear and bending moment diagrams for the beam and loading shown.

SOLUTION:

- Taking entire beam as a free-body, calculate reactions at *B* and *D*.
- Find equivalent internal force-couple systems for free-bodies formed by cutting beam on either side of load application points.
- Plot results.

From: Rabiei

Sample Problem 7.2



SOLUTION:

- Taking entire beam as a free-body, calculate reactions at *B* and *D*.
- Find equivalent internal force-couple systems at sections on either side of load application points.

$$\sum F_y = 0: \quad -20\,\text{kN} - V_1 = 0 \qquad \boxed{V_1 = -20\,\text{kN}}$$

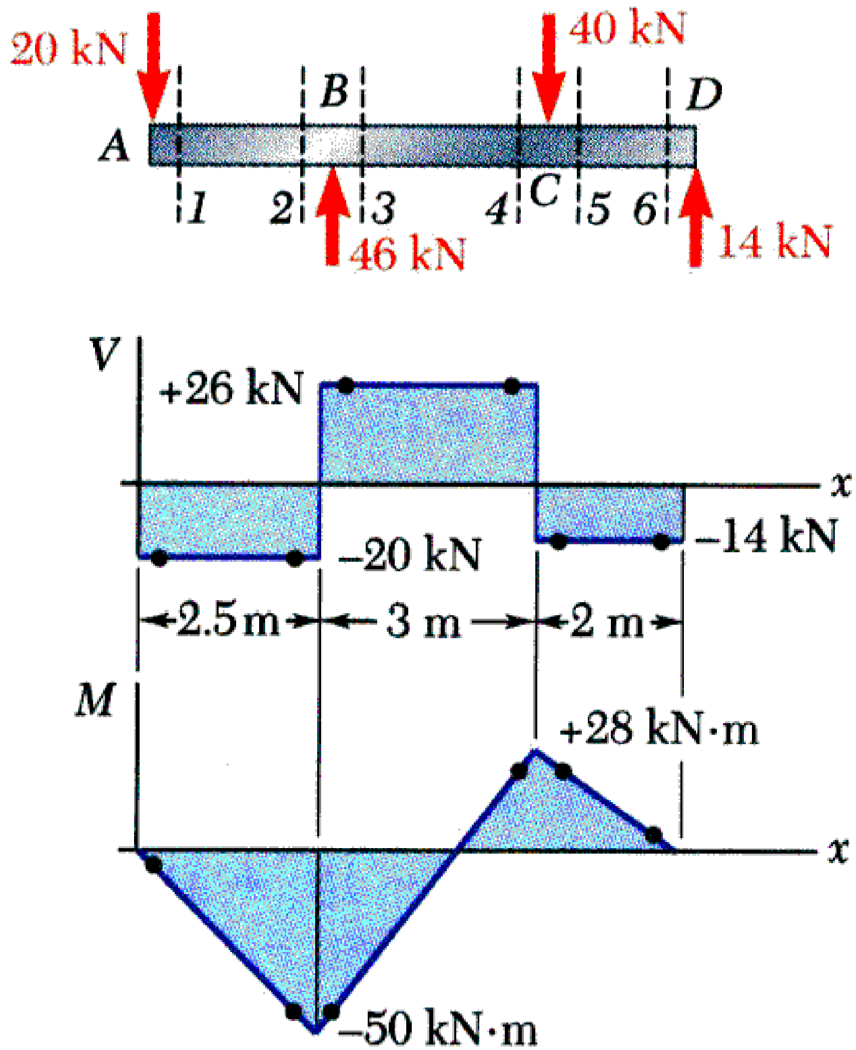
$$\sum M_2 = 0: \quad (20\,\text{kN})(0\,\text{m}) + M_1 = 0 \qquad \boxed{M_1 = 0}$$

Similarly,

$$\boxed{\begin{array}{ll} V_3 = 26\,\text{kN} & M_3 = -50\,\text{kN}\cdot\text{m} \\ V_4 = 26\,\text{kN} & M_4 = -50\,\text{kN}\cdot\text{m} \\ V_5 = 26\,\text{kN} & M_5 = -50\,\text{kN}\cdot\text{m} \\ V_6 = 26\,\text{kN} & M_6 = -50\,\text{kN}\cdot\text{m} \end{array}}$$

From: Rabiei

Sample Problem 7.2



- Plot results.

  Note that shear is of constant value between concentrated loads and bending moment varies linearly.

From: Rabiei