Fourth Edition

CHAPTER

2

# MECHANICS OF MATERIALS

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## Stress and Strain – Axial Loading

# Stress & Strain: Axial Loading

- Suitability of a structure or machine may depend on the deformations in the structure as well as the stresses induced under loading. Statics analyses alone are not sufficient.
- Considering structures as deformable allows determination of member forces and reactions which are statically indeterminate.
- Determination of the stress distribution within a member also requires consideration of deformations in the member.
- Chapter 2 is concerned with deformation of a structural member under axial loading. Later chapters will deal with torsional and pure bending loads.

# Normal Strain

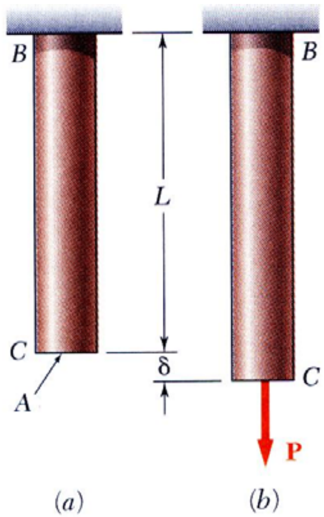



Fig. 2.1

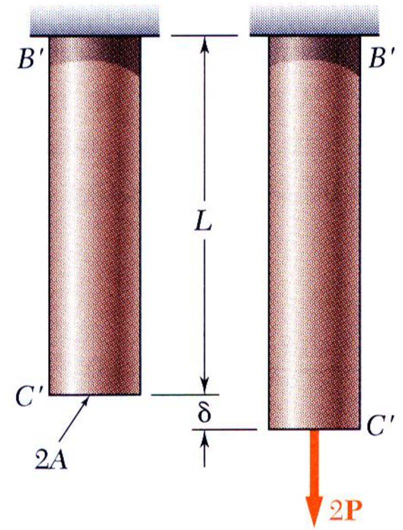



Fig. 2.3

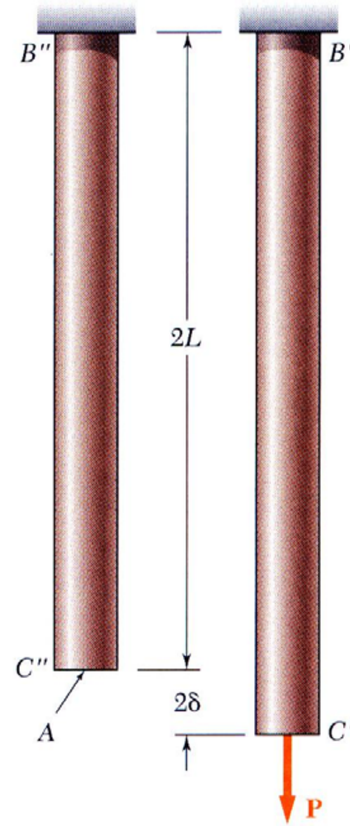



Fig. 2.4

$$\sigma = \frac{P}{A} = \text{stress}$$

$$\varepsilon = \frac{\delta}{L} = \text{normal strain}$$

$$\sigma = \frac{2P}{2A} = \frac{P}{A}$$

$$\varepsilon = \frac{\delta}{L}$$

$$\sigma = \frac{P}{A}$$

$$\varepsilon = \frac{2\delta}{2L} = \frac{\delta}{L}$$

# Stress-Strain Test

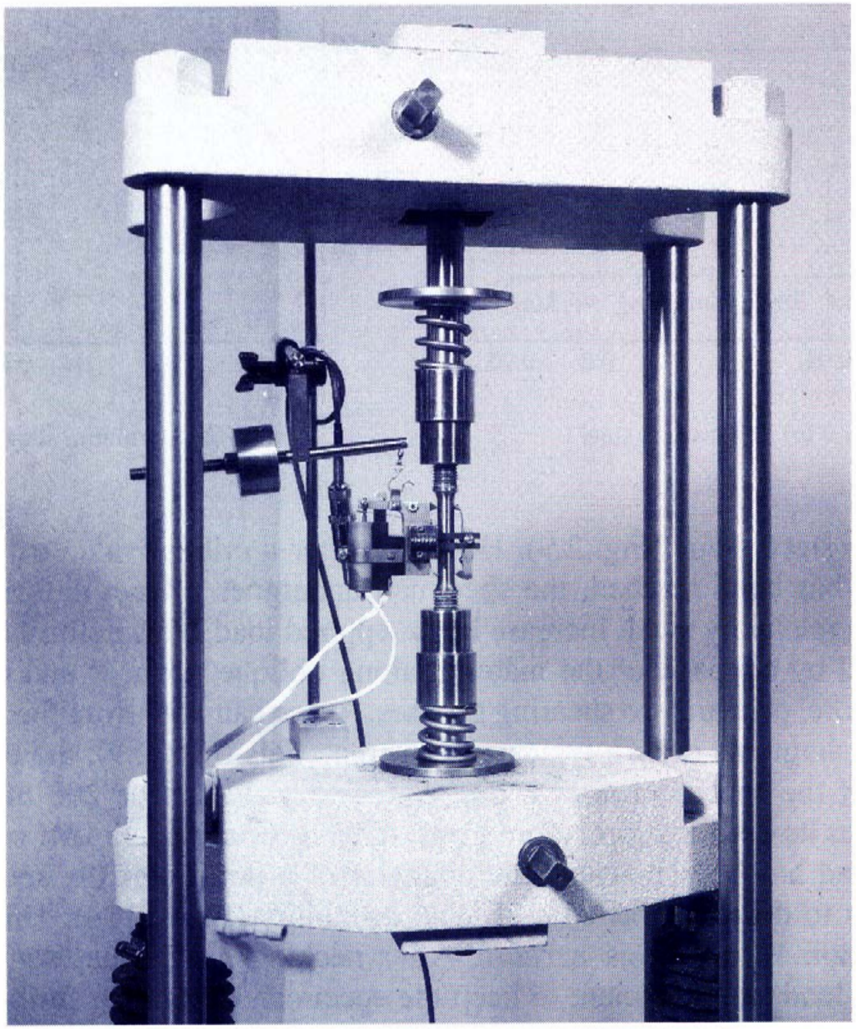


**Fig. 2.7** This machine is used to test tensile test specimens, such as those shown in this chapter.

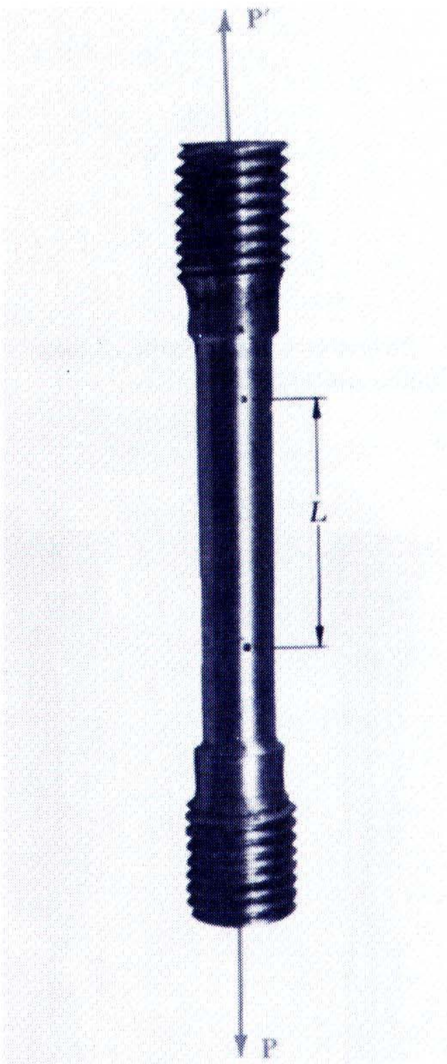
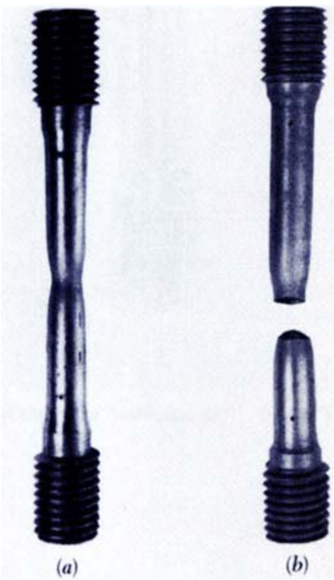


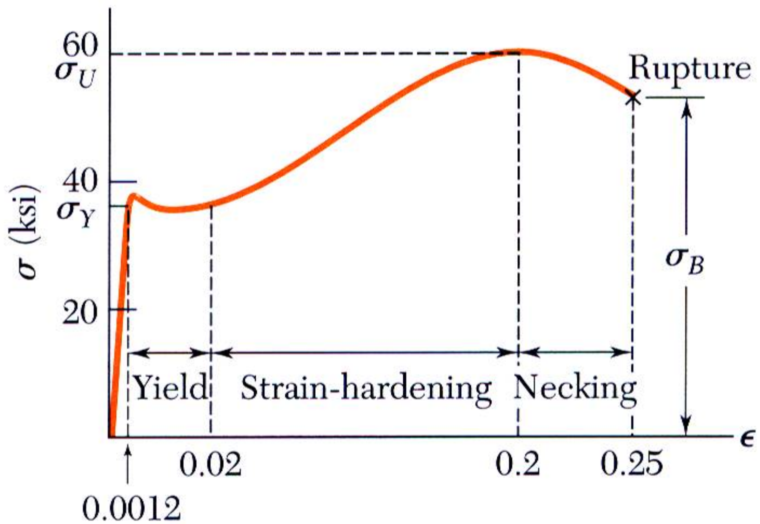

**Fig. 2.8** Test specimen with tensile load.

6

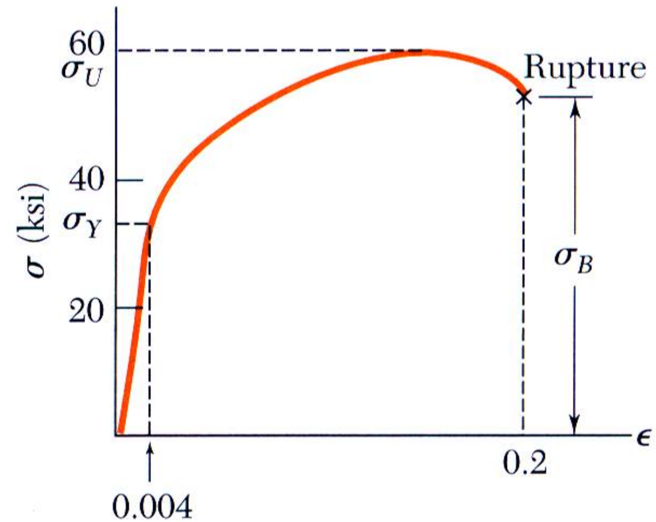
From: Rezaei, Chapter 2, Part 1

# Stress-Strain Diagram: Ductile Materials




(*a*) Low-carbon steel

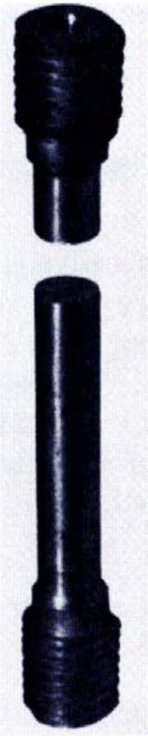



(*b*) Aluminum alloy



From: Rezaei, Chapter 2, Part 1

# Stress-Strain Diagram: Brittle Materials



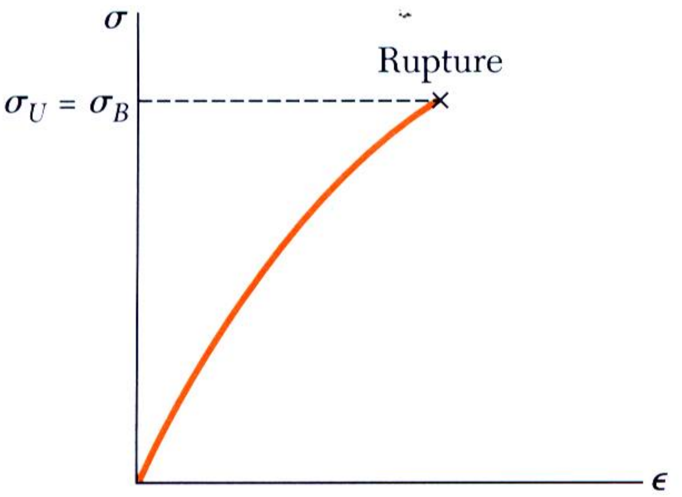



**Fig. 2.11** Stress-strain diagram for a typical brittle material.

# Hooke's Law: Modulus of Elasticity

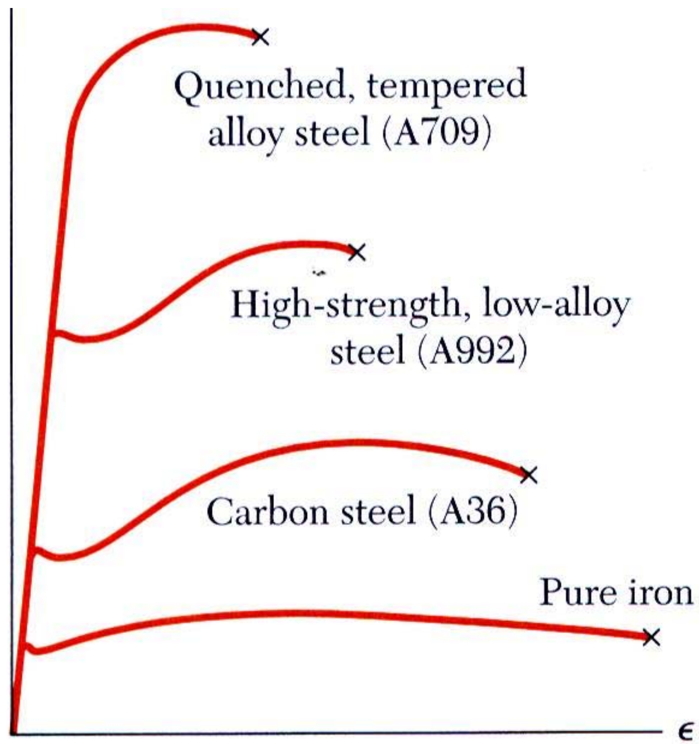



**Fig. 2.16** Stress-strain diagrams for iron and different grades of steel.

- Below the yield stress

$$\sigma = E\varepsilon$$

$$E = \text{Youngs Modulus or Modulus of Elasticity}$$

- Strength is affected by alloying, heat treating, and manufacturing process but stiffness (Modulus of Elasticity) is not.

# Elastic vs. Plastic Behavior

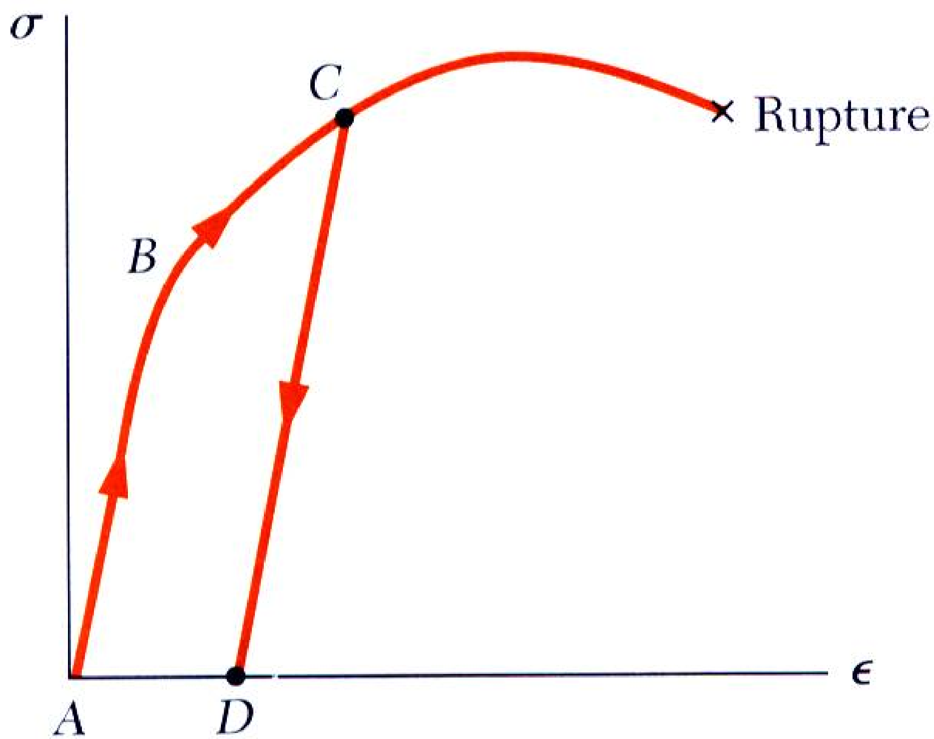



Fig. 2.18

- If the strain disappears when the stress is removed, the material is said to behave *elastically*.
- The largest stress for which this occurs is called the *elastic limit*.
- When the strain does not return to zero after the stress is removed, the material is said to behave *plastically*.

# Deformations Under Axial Loading

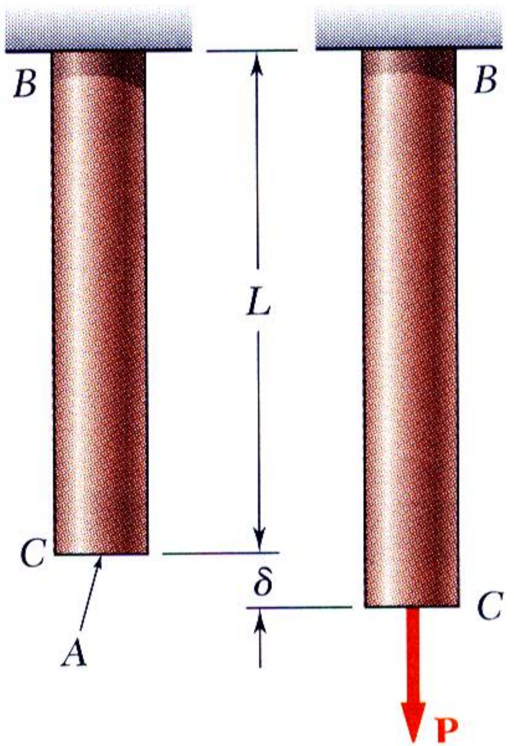



Fig. 2.22

- From Hooke's Law:

$$\sigma = E\varepsilon \qquad \varepsilon = \frac{\sigma}{E} = \frac{P}{AE}$$

- From the definition of strain:

$$\varepsilon = \frac{\delta}{L}$$

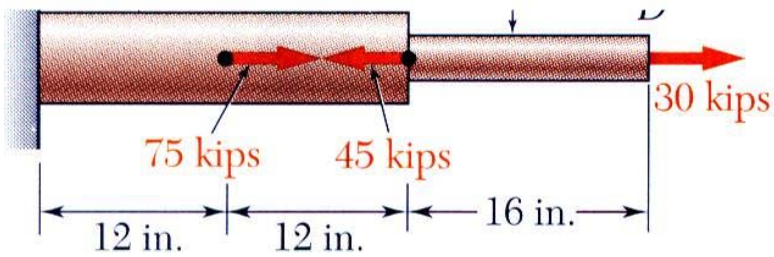
- Equating and solving for the deformation,

$$\delta = \frac{PL}{AE}$$

- With variations in loading, cross-section or material properties,

$$\delta = \sum_i \frac{P_i L_i}{A_i E_i}$$

# Example 2.01




$$E = 29\times10^{-6}\,\text{psi}$$

$$D = 1.07\,\text{in.}\quad d = 0.618\,\text{in.}$$

Determine the deformation of the steel rod shown under the given loads.

SOLUTION:

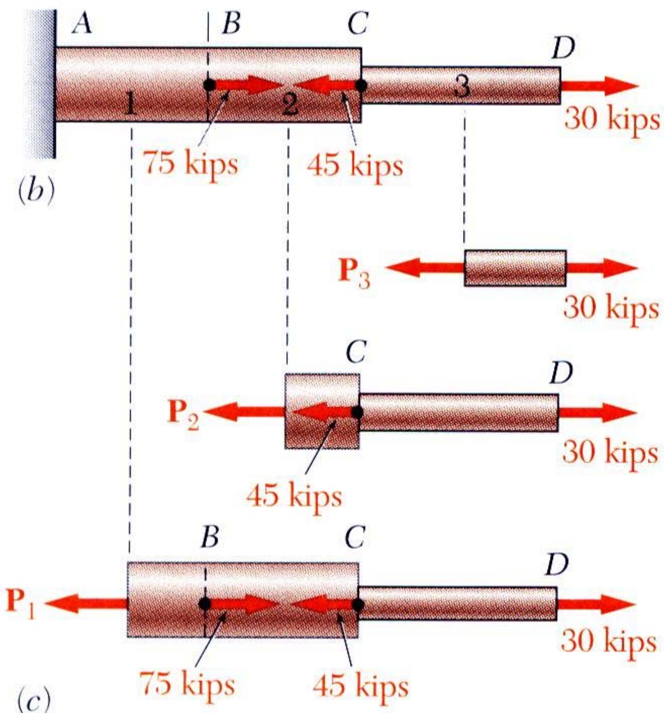
- Divide the rod into components at the load application points.
- Apply a free-body analysis on each component to determine the internal force
- Evaluate the total of the component deflections.



From: Rezaei, Chapter 2, Part 1

SOLUTION:

- Divide the rod into three components:




$$L_1 = L_2 = 12\text{ in.} \qquad L_3 = 16\text{ in.}$$

$$A_1 = A_2 = 0.9\text{ in}^2 \qquad A_3 = 0.3\text{ in}^2$$

- Apply free-body analysis to each component to determine internal forces,

$$P_1 = 60\times10^3\text{lb}$$

$$P_2 = -15\times10^3\text{lb}$$

$$P_3 = 30\times10^3\text{lb}$$

- Evaluate total deflection,

$$\delta = \sum_i \frac{P_i L_i}{A_i E_i} = \frac{1}{E}\left(\frac{P_1 L_1}{A_1} + \frac{P_2 L_2}{A_2} + \frac{P_3 L_3}{A_3}\right)$$

$$= \frac{1}{29\times10^6}\left[\frac{\left(60\times10^3\right)12}{0.9} + \frac{\left(-15\times10^3\right)12}{0.9} + \frac{\left(30\times10^3\right)16}{0.3}\right]$$

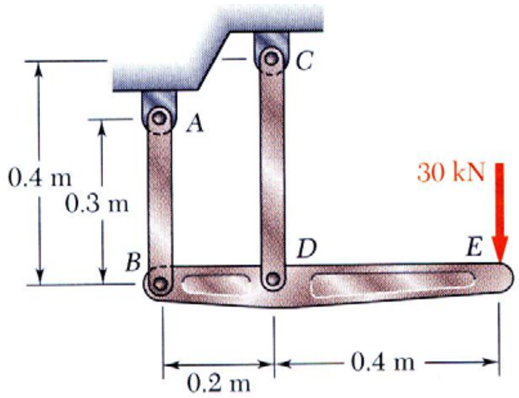
$$= 75.9\times10^{-3}\text{in.}$$

$$\boxed{\delta = 75.9\times10^{-3}\text{ in.}}$$



From: Rezaei, Chapter 2, Part 1

# Sample Problem 2.1




The rigid bar *BDE* is supported by two links *AB* and *CD*.

Link *AB* is made of aluminum (*E* = 70 GPa) and has a cross-sectional area of 500 mm$^2$. Link *CD* is made of steel (*E* = 200 GPa) and has a cross-sectional area of (600 mm$^2$).

For the 30-kN force shown, determine the deflection a) of *B*, b) of *D*, and c) of *E*.

SOLUTION:

- Apply a free-body analysis to the bar *BDE* to find the forces exerted by links *AB* and *DC*.
- Evaluate the deformation of links *AB* and *DC* or the displacements of *B* and *D*.
- Work out the geometry to find the deflection at E given the deflections at B and D.

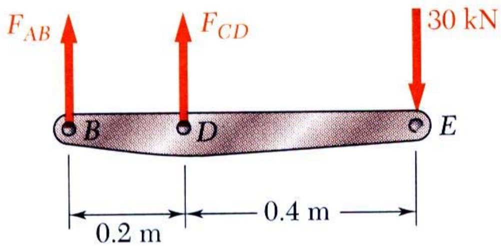


From: Rezaei, Chapter 2, Part 1

# Sample Problem 2.1

SOLUTION:

Free body: Bar *BDE*




$$+\circlearrowleft \sum M_B = 0$$

$$0 = -(30\,\text{kN} \times 0.6\,\text{m}) + F_{CD} \times 0.2\,\text{m}$$

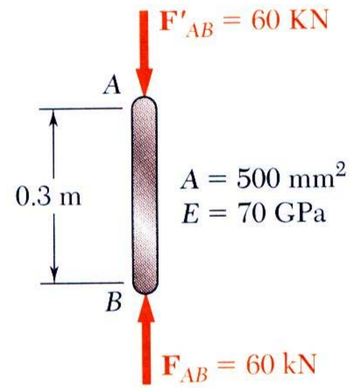
$$F_{CD} = +90\,\text{kN} \quad \textit{tension}$$

$$+\circlearrowleft \sum \text{M}_\text{D} = 0$$

$$0 = -(30\,\text{kN} \times 0.4\,\text{m}) - F_{AB} \times 0.2\,\text{m}$$

$$F_{AB} = -60\,\text{kN} \quad \textit{compression}$$

Displacement of *B*:



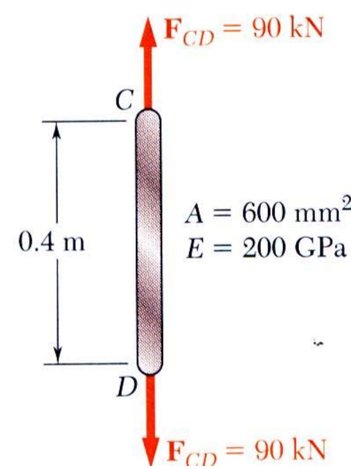

$$\delta_B = \frac{PL}{AE}$$

$$= \frac{(-60\times10^3\,\text{N})(0.3\,\text{m})}{(500\times10^{-6}\,\text{m}^2)(70\times10^9\,\text{Pa})}$$

$$= -514\times10^{-6}\,\text{m}$$

$$\boxed{\delta_B = 0.514\,\text{mm} \uparrow}$$

Displacement of *D*:




$$\delta_D = \frac{PL}{AE}$$

$$= \frac{(90\times10^3\,\text{N})(0.4\,\text{m})}{(600\times10^{-6}\,\text{m}^2)(200\times10^9\,\text{Pa})}$$

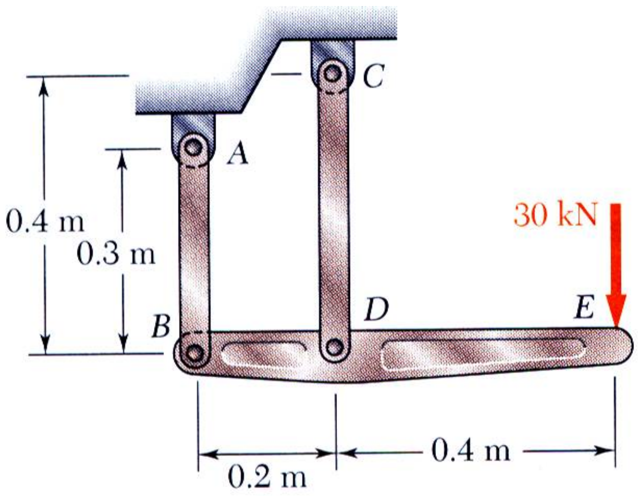
$$= 300\times10^{-6}\,\text{m}$$

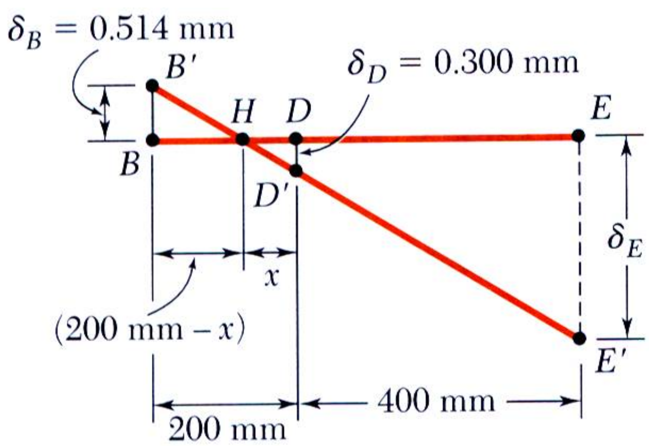
$$\boxed{\delta_D = 0.300\,\text{mm} \downarrow}$$



From: Rezaei, Chapter 2, Part 1

# Sample Problem 2.1




Displacement of D:

$$\frac{BB'}{DD'} = \frac{BH}{HD}$$

$$\frac{0.514\text{ mm}}{0.300\text{ mm}} = \frac{(200\text{ mm}) - x}{x}$$

$$x = 73.7\text{ mm}$$

$$\frac{EE'}{DD'} = \frac{HE}{HD}$$

$$\frac{\delta_E}{0.300\text{ mm}} = \frac{(400 + 73.7)\text{mm}}{73.7\text{ mm}}$$

$$\delta_E = 1.928\text{ mm}$$

$$\boxed{\delta_E = 1.928\text{ mm} \downarrow}$$



From: Rezaei, Chapter 2, Part 1