Fourth Edition

CHAPTER

1

MECHANICS OF MATERIALS

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Introduction – Concept of Stress

From: Rezaei

Concept of Stress

- The main objective of the study of the mechanics of materials is to provide the future engineer with the means of analyzing and designing various machines and load bearing structures.

- Both the analysis and design of a given structure involve the determination of *stresses* and *deformations*. This chapter is devoted to the concept of stress.

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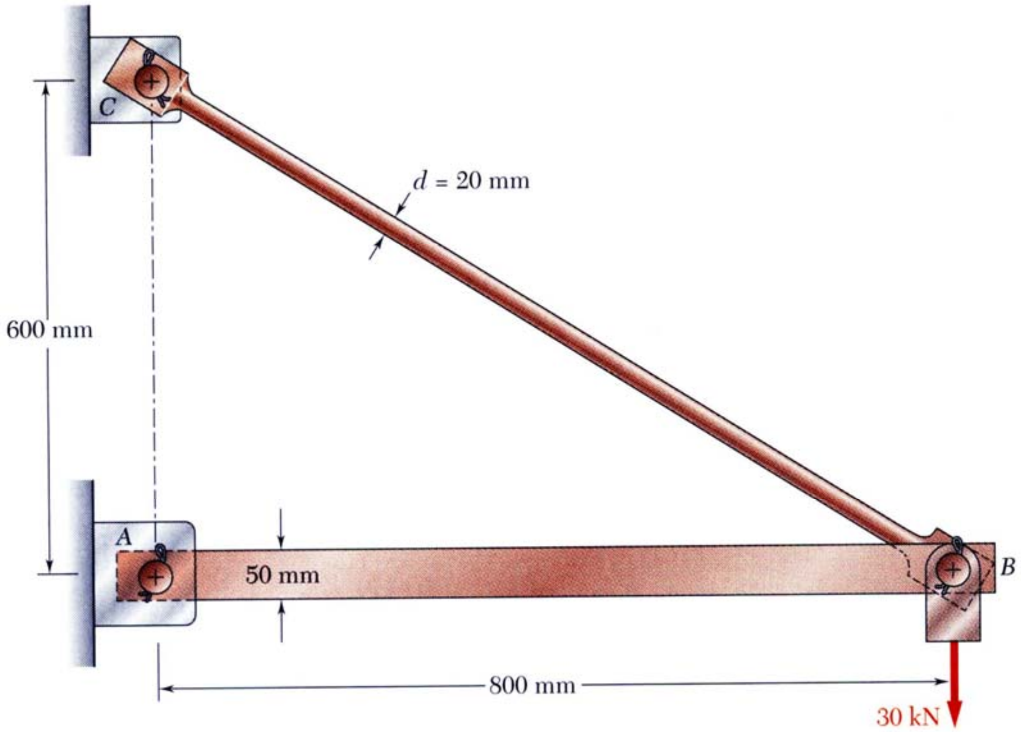
Review of Statics



- The structure is designed to support a 30 kN load
- The structure consists of a boom and rod joined by pins (zero moment connections) at the junctions and supports
- Perform a static analysis to determine the internal force in each structural member and the reaction forces at the supports

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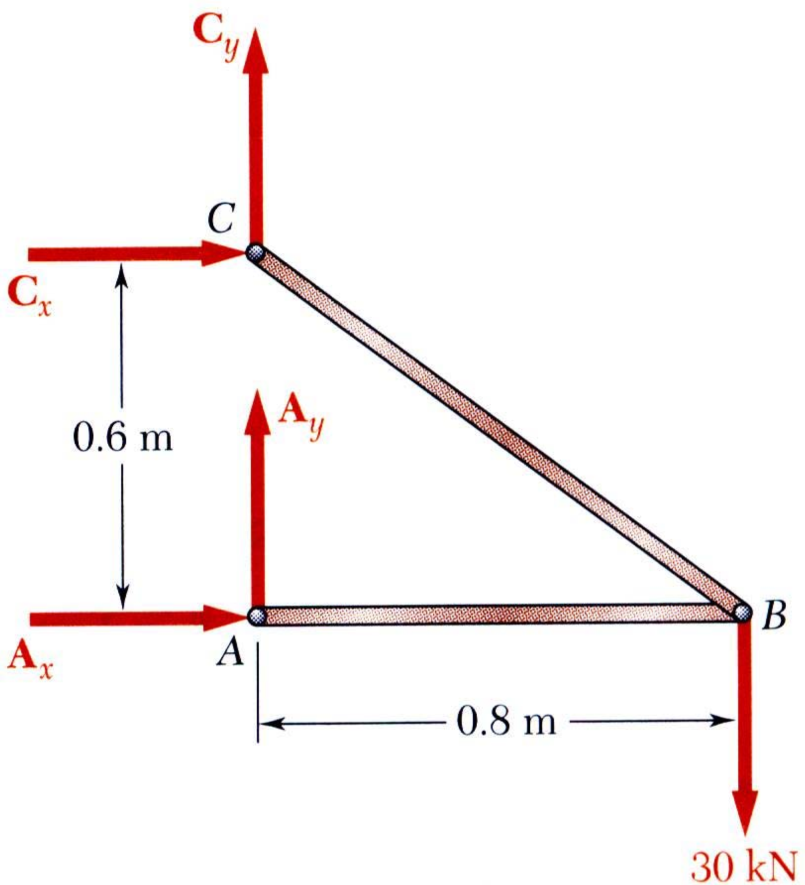
Structure Free-Body Diagram



- Structure is detached from supports and the loads and reaction forces are indicated
- Conditions for static equilibrium:

$$+\circlearrowleft \sum M_C = 0 = A_x(0.6\,\text{m}) - (30\,\text{kN})(0.8\,\text{m})$$

$$A_x = 40\,\text{kN}$$

$$\overset{+}{\rightarrow} \sum F_x = 0 = A_x + C_x$$

$$C_x = -A_x = -40\,\text{kN}$$

$$+\uparrow \sum F_y = 0 = A_y + C_y - 30\,\text{kN} = 0$$

$$A_y + C_y = 30\,\text{kN}$$

- A_y and C_y can not be determined from these equations

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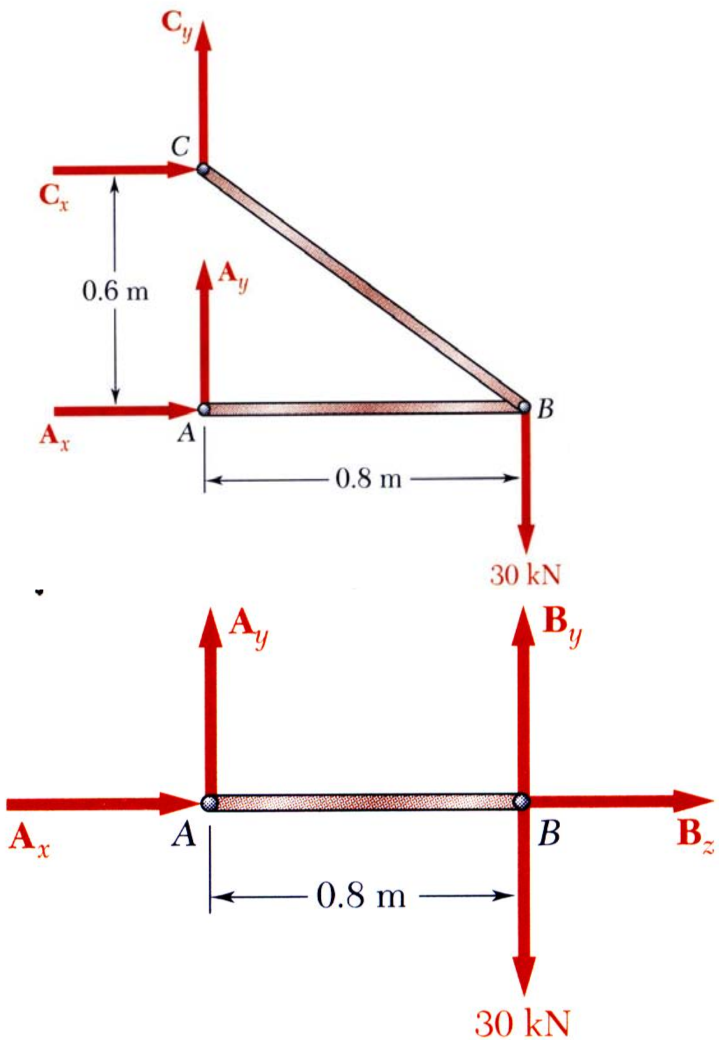
Component Free-Body Diagram



- In addition to the complete structure, each component must satisfy the conditions for static equilibrium
- Consider a free-body diagram for the boom:

$$+\circlearrowleft \sum M_B = 0 = -A_y(0.8\,\text{m})$$

$$A_y = 0$$

substitute into the structure equilibrium equation

$$C_y = 30\,\text{kN}$$

- Results:

$$A = 40\,\text{kN} \rightarrow \quad C_x = 40\,\text{kN} \leftarrow \quad C_y = 30\,\text{kN} \uparrow$$

Reaction forces are directed along boom and rod

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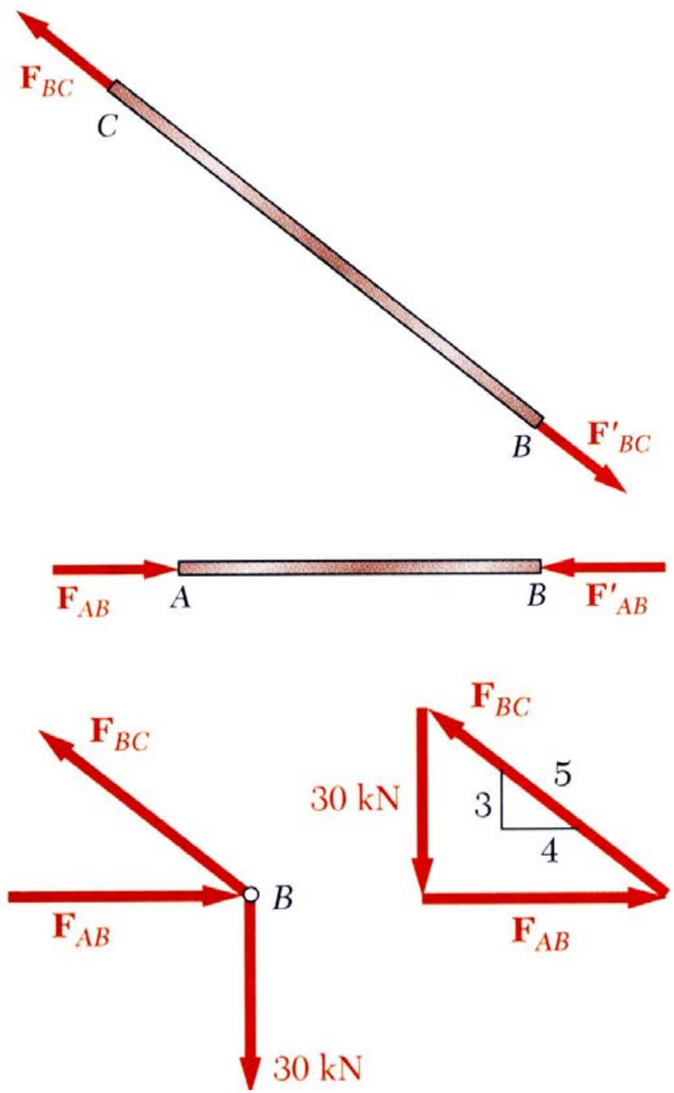
Method of Joints



- The boom and rod are 2-force members, i.e., the members are subjected to only two forces which are applied at member ends
- For equilibrium, the forces must be parallel to to an axis between the force application points, equal in magnitude, and in opposite directions
- Joints must satisfy the conditions for static equilibrium which may be expressed in the form of a force triangle:

$$\sum \vec{F}_B = 0$$

$$\frac{F_{AB}}{4} = \frac{F_{BC}}{5} = \frac{30\,\text{kN}}{3}$$

$$F_{AB} = 40\,\text{kN} \qquad F_{BC} = 50\,\text{kN}$$

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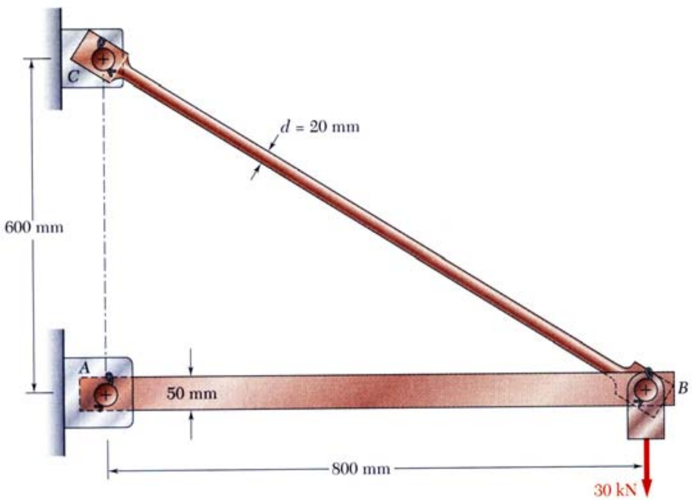
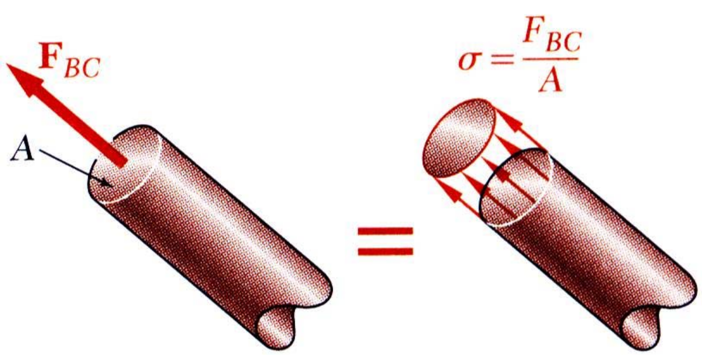
Stress Analysis



$d_{BC} = 20$ mm



Can the structure safely support the 30 kN load?

- From a statics analysis

  $F_{AB} = 40$ kN (compression)

  $F_{BC} = 50$ kN (tension)

- At any section through member BC, the internal force is 50 kN with a force intensity or stress of

$$\sigma_{BC} = \frac{P}{A} = \frac{50 \times 10^3 \text{ N}}{314 \times 10^{-6} \text{ m}^2} = 159 \text{ MPa}$$

- From the material properties for steel, the allowable stress is

$$\sigma_{\text{all}} = 165 \text{ MPa}$$

- Conclusion: the strength of member *BC* is adequate

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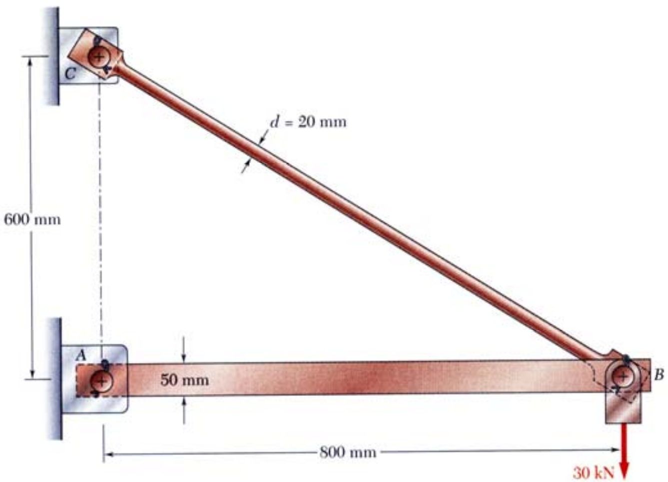
Design



- Design of new structures requires selection of appropriate materials and component dimensions to meet performance requirements

- For reasons based on cost, weight, availability, etc., the choice is made to construct the rod from aluminum (σ_{all}= 100 MPa). What is an appropriate choice for the rod diameter?

$$\sigma_{all} = \frac{P}{A} \qquad A = \frac{P}{\sigma_{all}} = \frac{50\times10^{3}\,\text{N}}{100\times10^{6}\,\text{Pa}} = 500\times10^{-6}\,\text{m}^2$$

$$A = \pi\frac{d^2}{4}$$

$$d = \sqrt{\frac{4A}{\pi}} = \sqrt{\frac{4\left(500\times10^{-6}\,\text{m}^2\right)}{\pi}} = 2.52\times10^{-2}\,\text{m} = 25.2\,\text{mm}$$

- An aluminum rod 26 mm or more in diameter is adequate

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